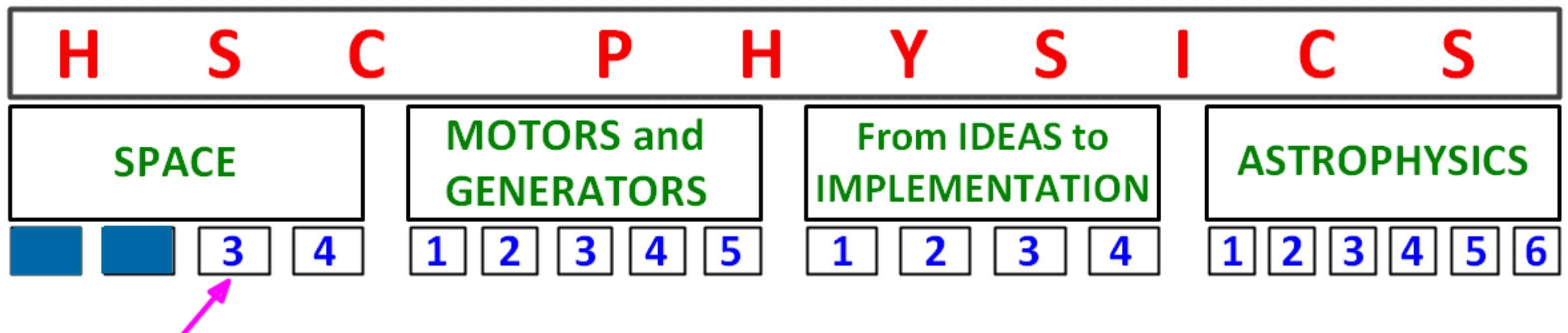


SPACE

1st Quarter; Module 1

PERIOD 18

Gravitational Field and Slingshot Effect



$$F = G \frac{m_1 \cdot m_2}{d^2}$$

m_1 : mass of the first object

m_2 : mass of the 2nd object

d : distance b/w 1st & 2nd objects
(centre to centre)

G = Un. gravitational constant = $6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$

the force exerted on
1st by 2nd &

the force exerted on
2nd by 1st are

* equal in mag.

* opp. in dir.

COMPLETE THE TABLE BELOW

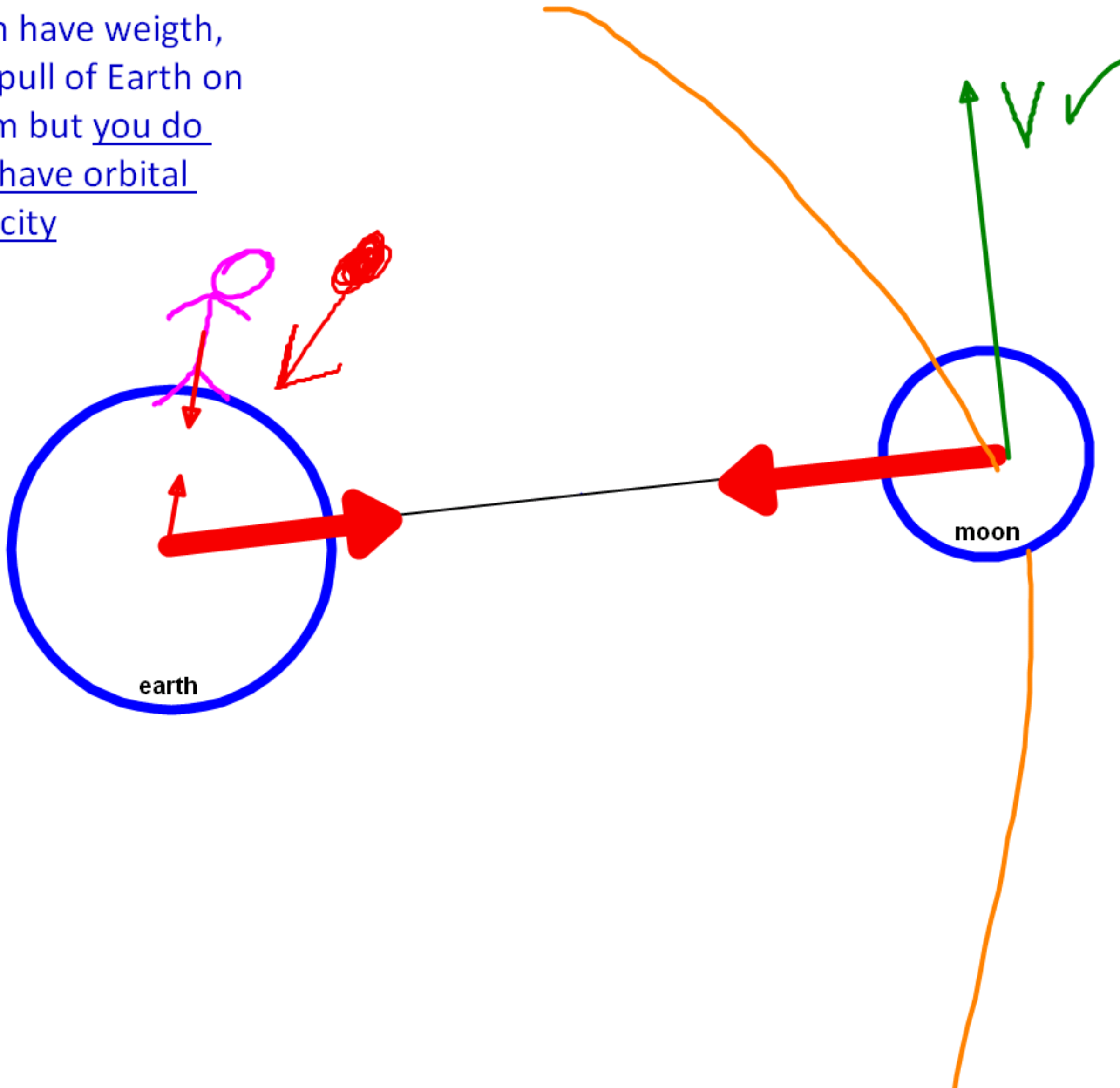
m_1	m_2	d	F
m	M	r	F
m	M	$r/2$	$4F$
$2m$	M	r	$2F$
$2m$	$3M$	r	$6F$
m	M	$2r$	$1/4 F$
m	M	$3r$	$1/9 F$
m	M	$\frac{1}{10} r$	$100F$
$\frac{m}{2}$	$5M$	$2.5r$	$\frac{1}{2} \times 5 \times \frac{1}{2.5^2}$
$3m$	$\frac{2}{5} M$	$\frac{1}{3} r$	$3 \times \frac{2}{5} \times \frac{1}{\frac{1}{3}^2}$

~~63~~ $\frac{54}{5}$

WHAT IS THE DIFFERENCE BETWEEN YOU AND THE MOON (IN TERMS OF MOTION)?

Both have weight,
the pull of Earth on
them but you do
not have orbital
velocity

Both have weight,
the pull of Earth on
them, but moon
does have orbital
velocity to stay in
the orbit



Worked example 3.1B

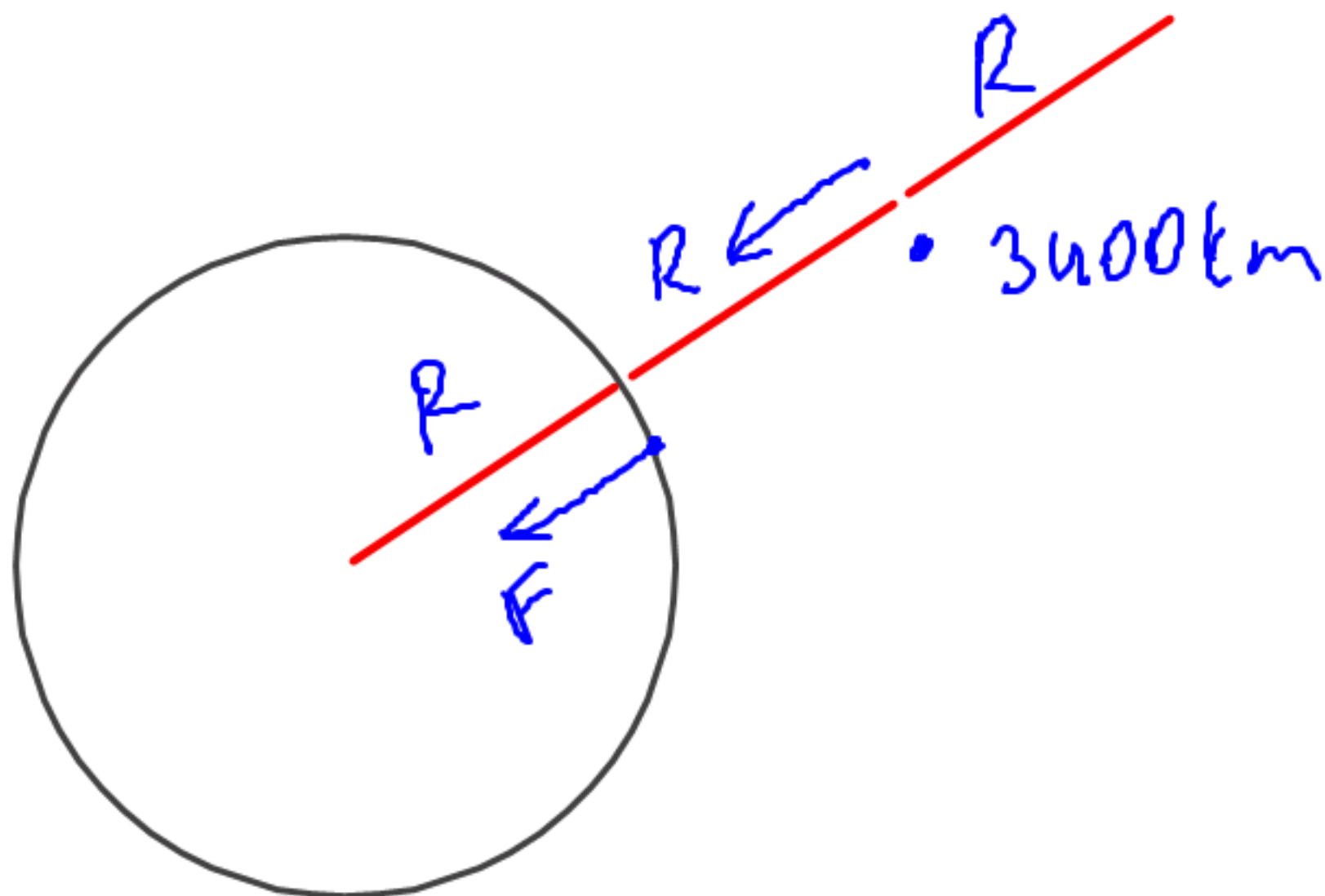
The gravitational force that acts on a 1200 kg space probe on the surface of Mars is $4.43 \times 10^3 \text{ N}$. The radius of Mars is 3400 km. Without using the mass of Mars, determine the gravitational force that acts on the space probe when it is:

- a 3400 km above the surface of Mars
- b 6800 km above the surface of Mars.

$$4.43 \times 10^3 = 6 \times \frac{M \times 1200}{(3.4 \times 10^6)^2}$$

$$F_{\text{at } 3400 \text{ km surf}} = F/4$$

$$F_{\text{at } 6800 \text{ km surf}} = \frac{1}{9} F$$

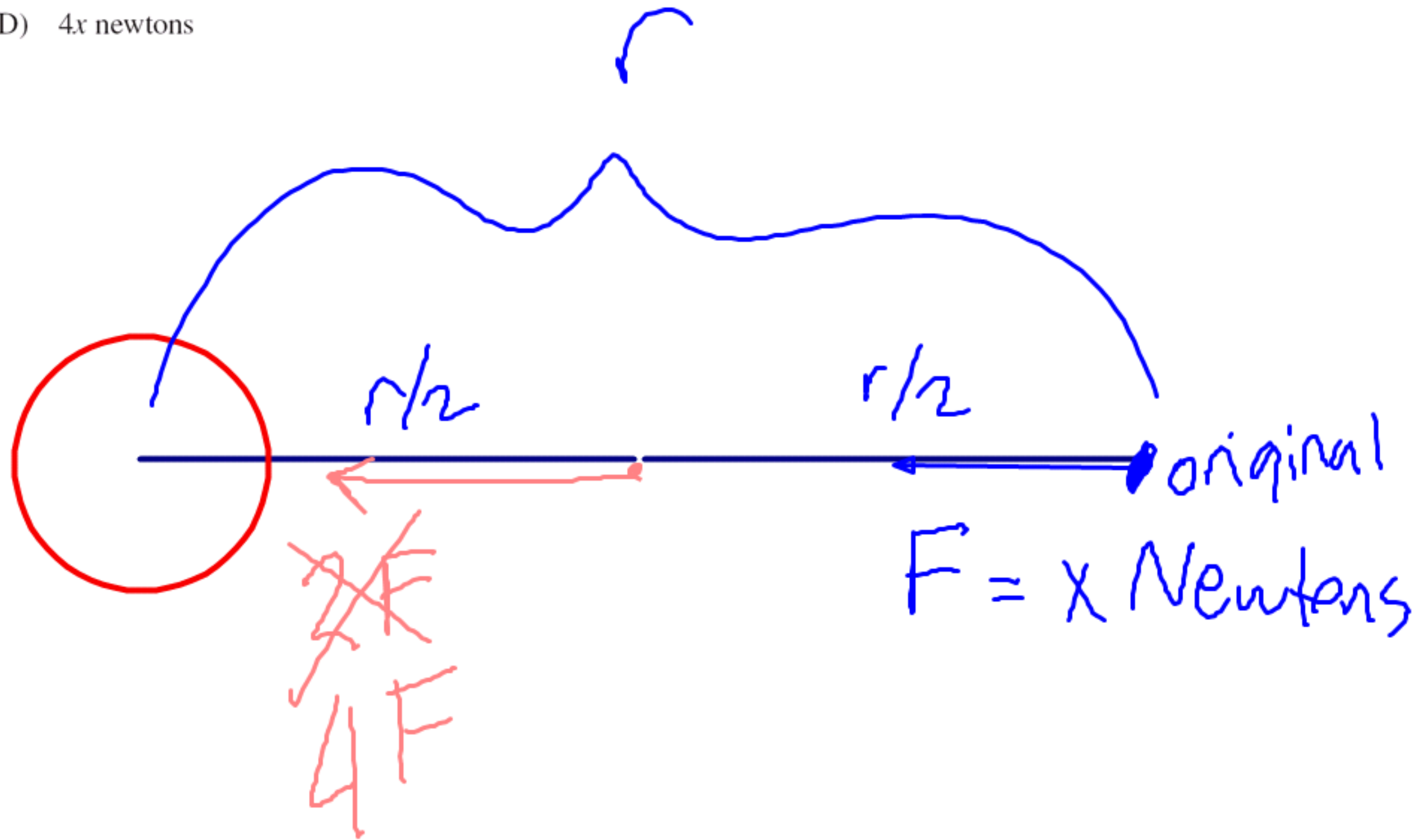


2004 HSC QUESTION

- 3 A spaceship at a distance r metres from the centre of a star experiences a gravitational force of x newtons. The spaceship moves a distance $\frac{r}{2}$ towards the star.

What is the gravitational force acting on the spaceship when it is at this new location?

- (A) $\frac{x}{2}$ newtons
- (B) x newtons
- (C) $2x$ newtons
- (D) $4x$ newtons



HOMEWORK (DUE TOMORROW)

For these questions, assume that $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, and that distances are measured between the centres of the bodies.

1 For this question, assume that:

mass of Earth = $6.0 \times 10^{24} \text{ kg}$

radius of Earth = $6.4 \times 10^6 \text{ m}$

mass of the Moon = $7.3 \times 10^{22} \text{ kg}$

mean radius of Moon's orbit = $3.8 \times 10^8 \text{ m}$

Calculate the gravitational force of attraction that exists between the following objects.

- a** A 100 g apple and a 200 g orange that are 50 cm apart
- b** A pair of 1000 kg boulders that are 10.0 m apart
- c** The Earth and a satellite of mass 20 000 kg in orbit at an altitude of 600 km
- d** The Moon and the Earth
- e** The Earth and a 60 kg student standing on its surface
- f** A proton of mass $1.67 \times 10^{-27} \text{ kg}$ and an electron of mass $9.11 \times 10^{-31} \text{ kg}$ separated by $5.30 \times 10^{-11} \text{ m}$ in a hydrogen atom

5 A comet of mass 1000 kg is plummeting towards Jupiter. Jupiter has a mass of $1.90 \times 10^{27} \text{ kg}$ and a planetary radius of $7.15 \times 10^7 \text{ m}$. If the comet is about to crash into Jupiter, calculate the following:

- a** the magnitude of the gravitational force that Jupiter exerts on the comet
- b** the magnitude of the gravitational force that the comet exerts on Jupiter
- c** the acceleration of the comet towards Jupiter
- d** the acceleration of Jupiter towards the comet.

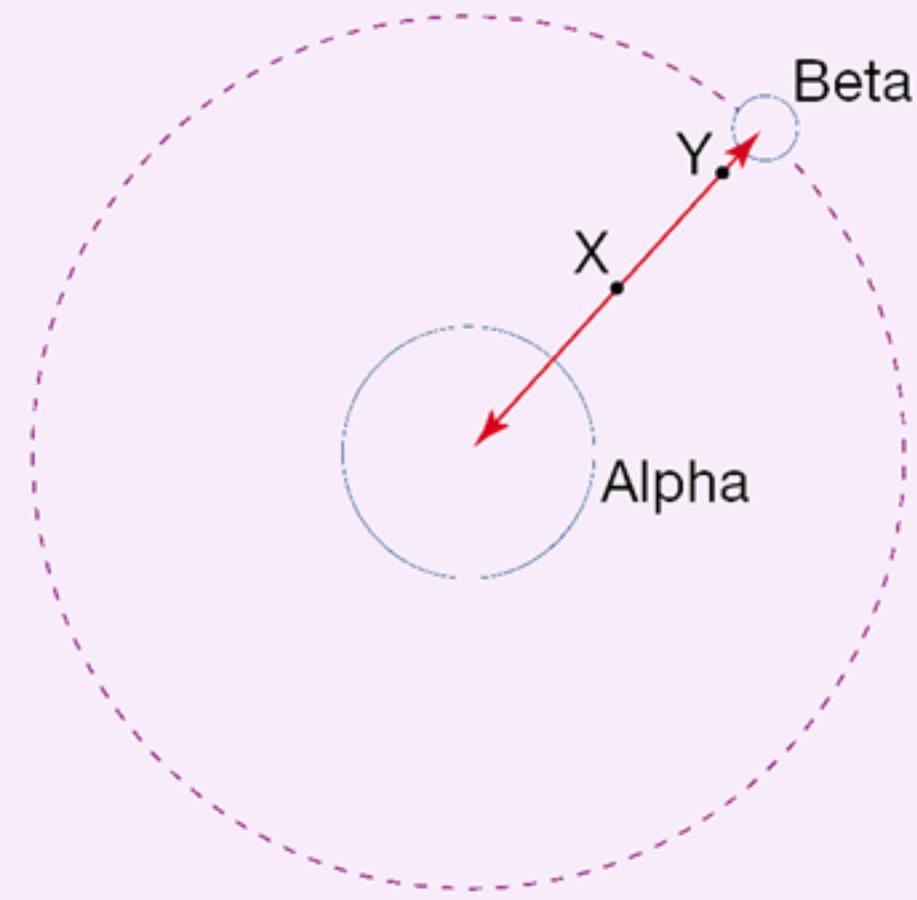
6 An astronaut travels away from Earth to a region in space where the gravitational force due to Earth is only 1.0% of that at Earth's surface. What distance, in Earth radii, is the astronaut from the centre of the Earth?

The following information applies to questions 7 and 8. The planet Alpha, whose mass is M , has one moon Beta of mass $0.01M$. The mean distance between the centres of Alpha and Beta is R .



hydrogen atom

- 2 Newton's law of universal gravitation defines the symbol G as a 'universal constant'.
- Explain the meaning of this term and its significance.
 - Identify another universal constant.
- 3 An astronaut standing on the surface of the Moon experiences a gravitational force of attraction of 160 N. He then moves away from the surface of the Moon to an altitude where the gravitational force is 40 N.
- How far from the centre of the Moon is this new location in terms of the radius of the Moon?
 - The astronaut now travels to another location at a height of three Moon radii above the surface. Calculate the gravitational force at this altitude.
- 4 Mars has two natural satellites. Phobos has a mean orbital radius of 9.4×10^6 m, while the other moon, Deimos, is located at a mean distance of 2.35×10^7 m from the centre of Mars. Data: mass of Mars = 6.42×10^{23} kg; mass of Phobos = 1.08×10^{16} kg; mass of Deimos = 1.8×10^{15} kg.
Calculate the value of the ratio:
$$\frac{\text{gravitational force exerted by Mars on Phobos}}{\text{gravitational force exerted by Mars on Deimos}}$$



- 7 a If an asteroid is at point X, exactly halfway between the centres of Alpha and Beta, calculate the value of the ratio:
- $$\frac{\text{force exerted on asteroid by Alpha}}{\text{force exerted on asteroid by Beta}}$$
- b At what distance, expressed in terms of R , from the planet Alpha, along a straight line joining the centres of Alpha and Beta, will the ratio expressed in part a be equal to 8100?
- 8 Point Y represents the distance from planet Alpha where the magnitude of the net gravitational force is zero. What is this distance in terms of R ?

SOLUTIONS OF “PERIOD 17” QUESTIONS

A1.

- a** Gravitational force $F = GMm/R^2$
 $= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(0.100 \text{ kg})(0.200 \text{ kg})/(50 \times 10^{-2} \text{ m})^2$
 $= 5.3 \times 10^{-12} \text{ N}$
- b** Gravitational force F
 $= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1000 \text{ kg})(1000 \text{ kg})/(10 \text{ m})^2$
 $= 6.7 \times 10^{-7} \text{ N}$
- c** Gravitational force F
 $= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(2.0 \times 10^4 \text{ kg})(6.0 \times 10^{24} \text{ kg})/(7.0 \times 10^6 \text{ m})^2$
 $= 1.6 \times 10^5 \text{ N}$
- d** Gravitational force F
 $= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(7.3 \times 10^{22} \text{ kg})(6.0 \times 10^{24} \text{ kg})/(3.8 \times 10^8 \text{ m})^2$
 $= 2.0 \times 10^{20} \text{ N}$
- e** Gravitational force F
 $= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(6.0 \times 10^{24} \text{ kg})(60 \text{ kg})/(6.4 \times 10^6 \text{ m})^2$
 $= 5.9 \times 10^2 \text{ N}$
- f** Gravitational force F
 $= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1.67 \times 10^{-27} \text{ kg})(9.11 \times 10^{-31} \text{ kg})/(5.33 \times 10^{-11} \text{ m})^2$
 $= 3.61 \times 10^{-47} \text{ N}$

A2.

- a** The term *universal constant* refers to the fact that the value of G (i.e. $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$) has exactly the same value everywhere in the Universe. This means that Newton's law of gravitation is valid throughout the Universe.
- b** Another very famous universal constant is the speed of light in a vacuum.

A3.

- a** The gravitational force at any point is inversely proportional to R^2 . Since the new force is one-quarter of the previous force, the distance from the centre of the Moon has doubled and the height is one Moon radius above the surface.
- b** $F = 200 \text{ N}/4^2 = 200 \text{ N}/16 = 12.5 \text{ N}$

A4.

$$\begin{aligned} F_{\text{Phobos}}/F_{\text{Deimos}} &= \frac{M_{\text{P}}}{R_{\text{P}}^2} \div \frac{M_{\text{D}}}{R_{\text{D}}^2} \\ &= \frac{M_{\text{P}} R_{\text{D}}^2}{M_{\text{D}} R_{\text{P}}^2} \\ &= \frac{1.08 \times 10^{16} \times (2.35 \times 10^7)^2}{1.08 \times 10^{15} \times (9.4 \times 10^6)^2} \\ &= 38 \end{aligned}$$

A5.

- a** $F = GMm/R^2$
 $= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1.9 \times 10^{27} \text{ kg})(1000 \text{ kg})/(7.15 \times 10^7 \text{ m})^2 = 2.48 \times 10^4 \text{ N}$
- b** The magnitude of the gravitational force that the probe exerts on Jupiter is equal to the magnitude of the gravitational force that Jupiter exerts on the probe $= 2.48 \times 10^4 \text{ N}$.
- c** Acceleration $= F/m$
 $= 2.48 \times 10^4 \text{ N}/1000 \text{ kg} = 24.8 \text{ m s}^{-2}$
- d** Acceleration $= F/m$
 $= 2.48 \times 10^4 \text{ N}/1.9 \times 10^{27} \text{ kg} = 1.3 \times 10^{-23} \text{ m s}^{-2}$

A6.

$$F = GMm/R^2$$

Using the inverse square relationship between F and R , for the force to be 1% ($\frac{1}{100}$) of its strength at the surface, the distance must be 10 times greater; i.e. at 10 Earth radii ($10R_{\text{E}}$).

A7.

- a** $F_{\alpha}/F_{\beta} = M/0.01M = 100$
- b** Let x = the required distance
 Then let $100(R - x)^2/x^2 = 8100$
 and $x = 0.1R$

A8.

Let d = required distance
 Then $1/d^2 = 0.01/(R - d)^2$
 and $d = 0.909R$

REVISION OF FORMULAS.

$$F_p = -\frac{GMm}{r}$$

$$F_c = \frac{mv^2}{r}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$V_c = \frac{2\pi r}{T} \text{ (orbital velocity)}$$

$$F_g = G \frac{mM}{d^2}$$

$$V_{esc} = \sqrt{\frac{2GM}{r}}$$

$$\frac{1}{2}mv^2 + -\frac{GMm}{r} = 0$$

$$E_k + E_p = 0 \text{ at space}$$

$$V_{orb} = \sqrt{\frac{GM}{d}}$$

$$F_g = F_c$$

$$G \frac{mM}{d^2} = \frac{mv^2}{d}$$

$$V_{orb} = \frac{2\pi r}{T}$$

$$F_g = F_{ret}$$

$$G \frac{mM}{d^2} = m \cdot a$$

$$a = g = \frac{GM}{d^2}$$

$$F_c = F_{ret}$$

$$\frac{mv^2}{r} = m \cdot a$$

$$a = \frac{v^2}{r}$$

Space 2

The Solar System is held together by gravity.

Students learn to:

- describe a gravitational field in the region surrounding a massive object in terms of its effects on other masses in it
- define Newton's Law of Universal Gravitation

$$F = G \frac{m_1 m_2}{d^2}$$

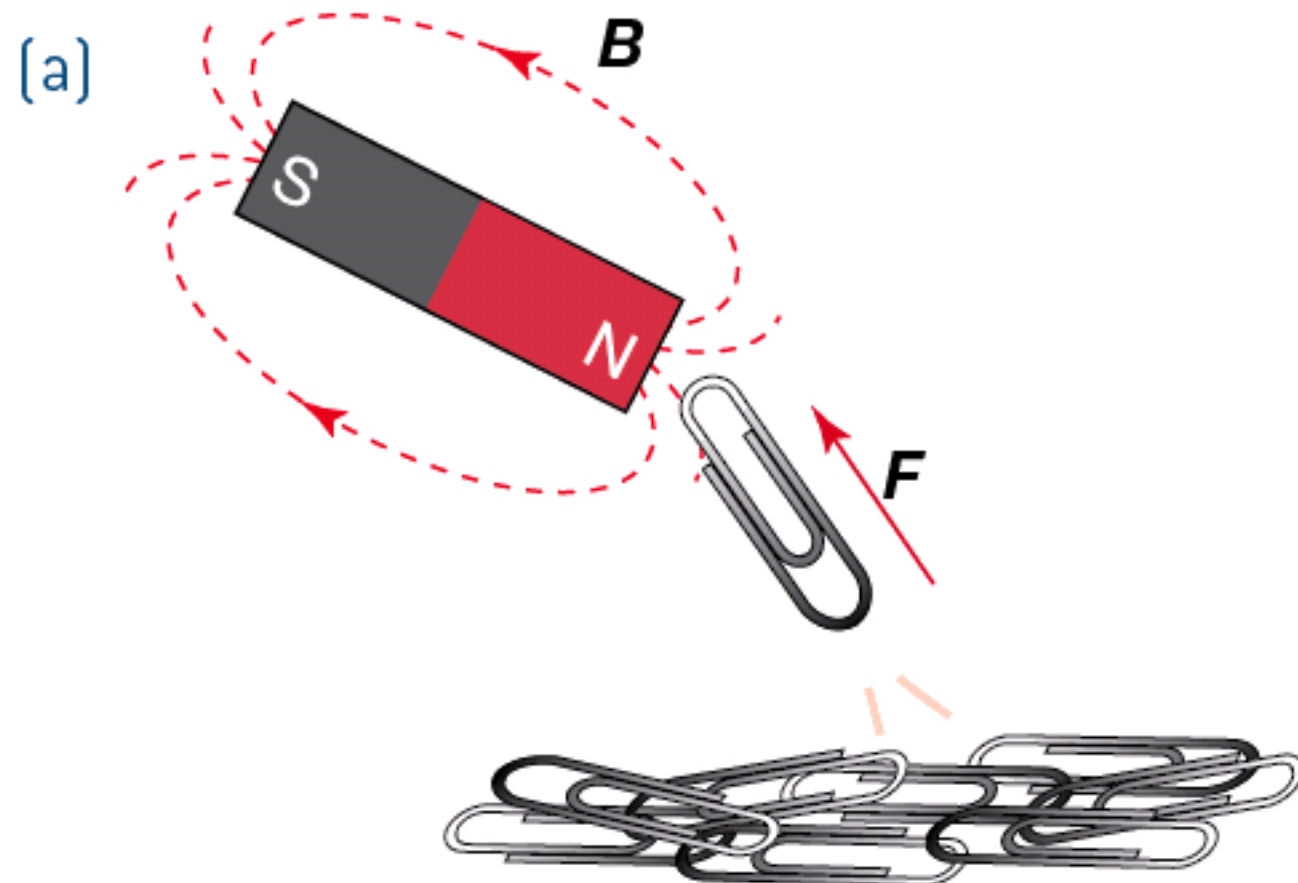
- discuss the importance of Newton's Law of Universal Gravitation in understanding and calculating the motion of satellites
- identify that a slingshot effect can be provided by planets for space probes

Students:

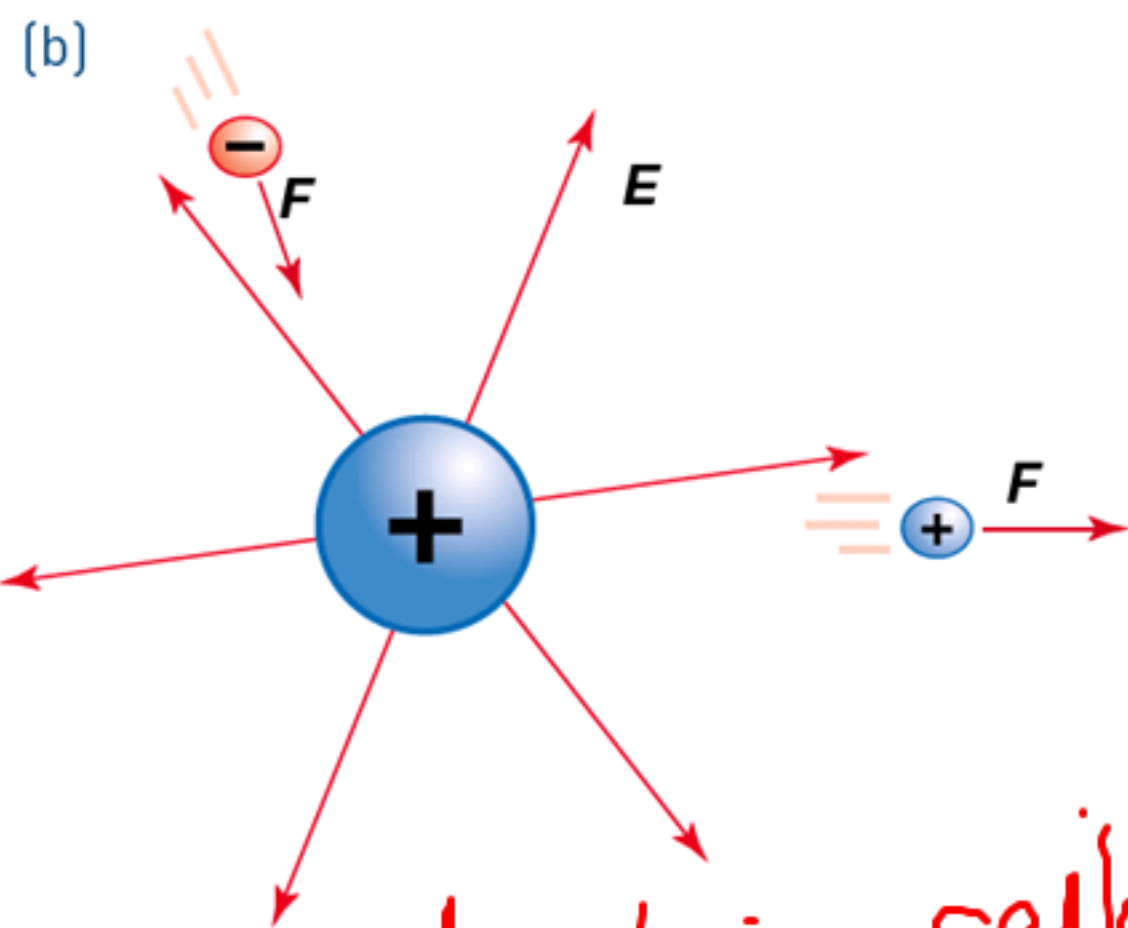
- present information and use available evidence to discuss the factors affecting the strength of the gravitational force
- solve problems and analyse information using

$$F = G \frac{m_1 m_2}{d}$$

FIELDS WE KNOW SO FAR



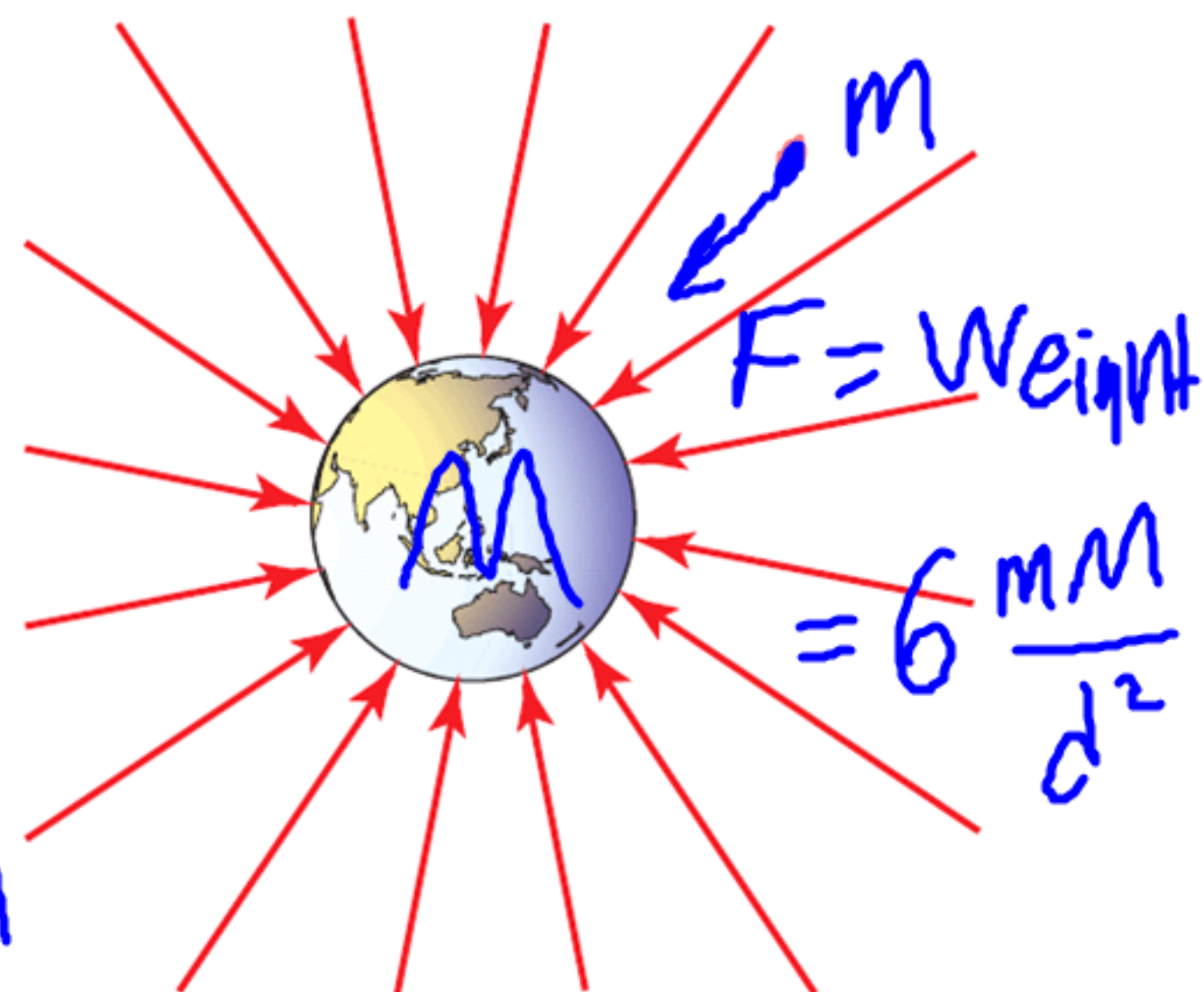
Magnetic field



electric field

g : gravitational field strength

(b) Around a planet



$$g = \frac{GM}{d^2}$$

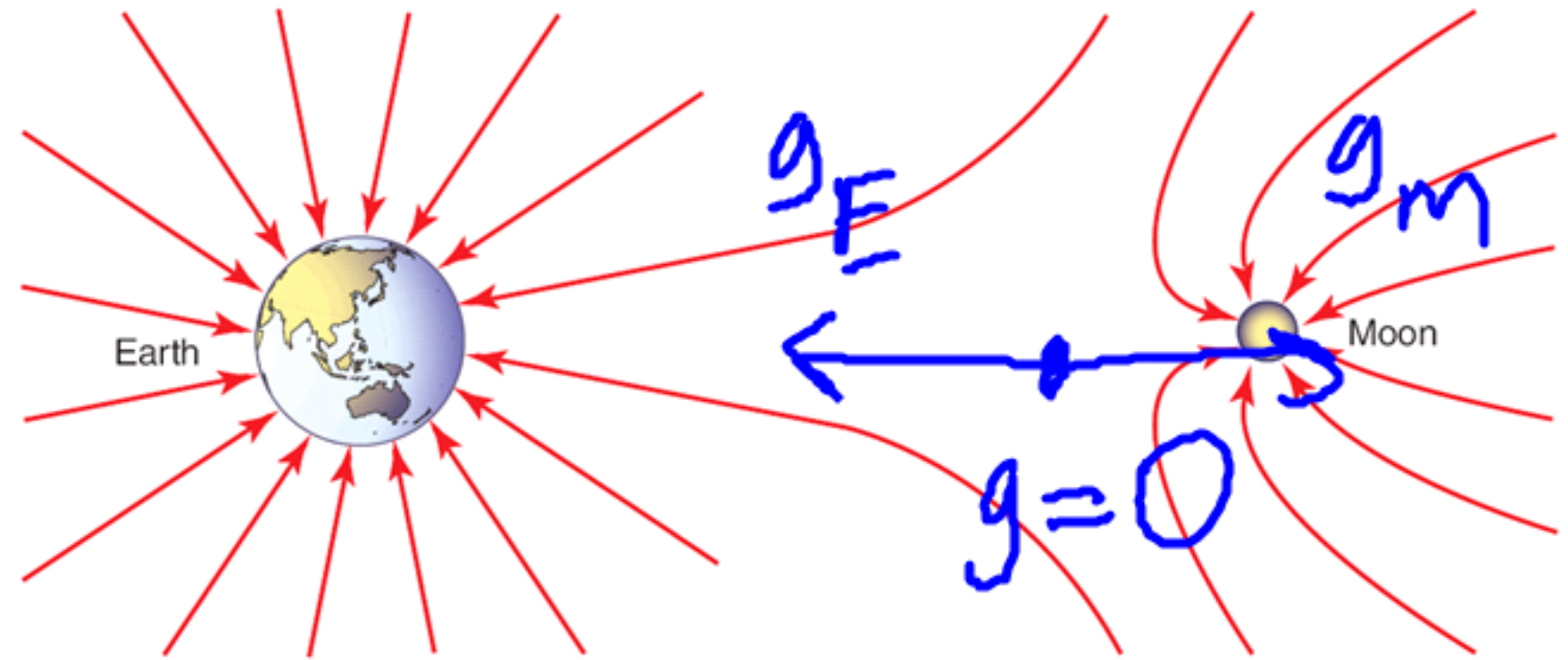
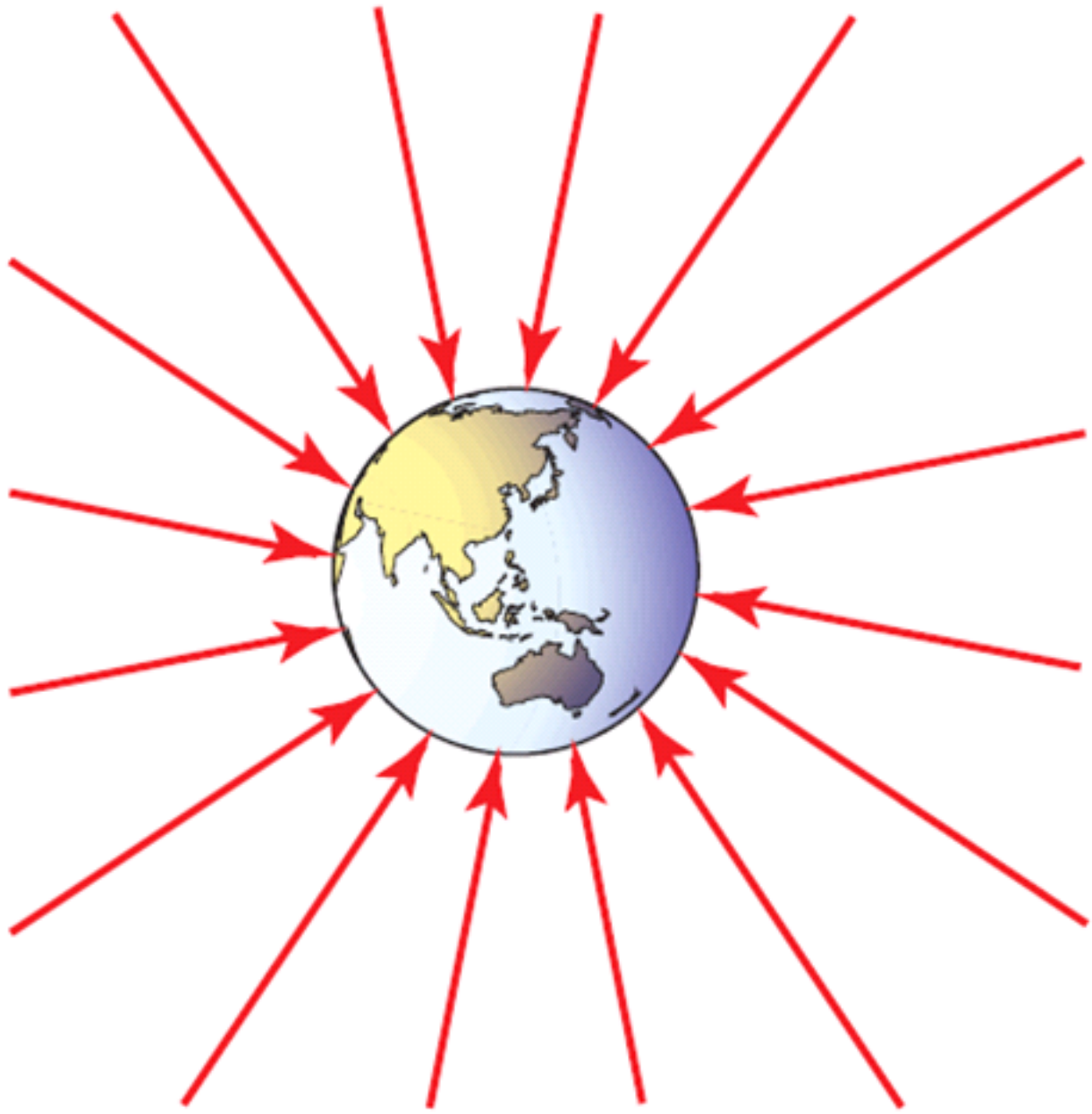
$$F = m \cdot g$$

$$= m \left(\frac{GM}{d^2} \right)$$

GRAVITATIONAL FIELD

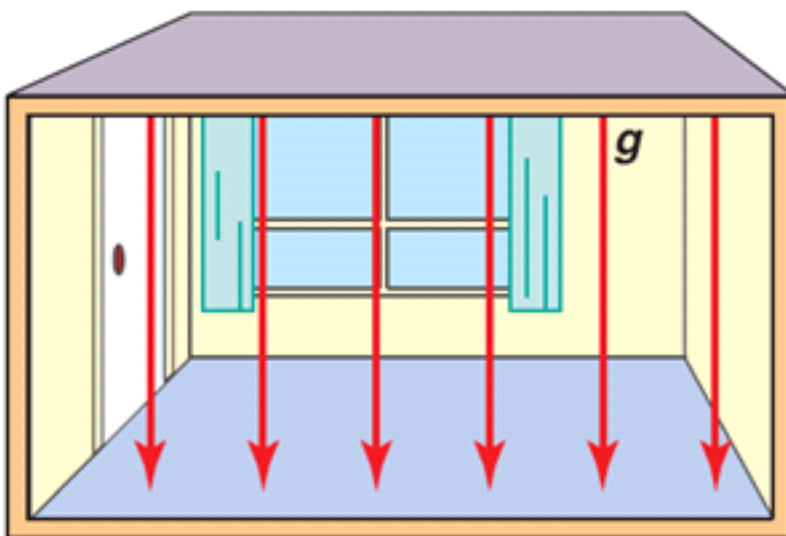
A **gravitational field** is a field within which any mass will experience a gravitational force.

(b) Around a planet



$$g_{\text{sum}} = 0$$

(a) In a room



field lines are parallel

HOW "g" OF EARTH CHANGES WITH ALTITUDE?

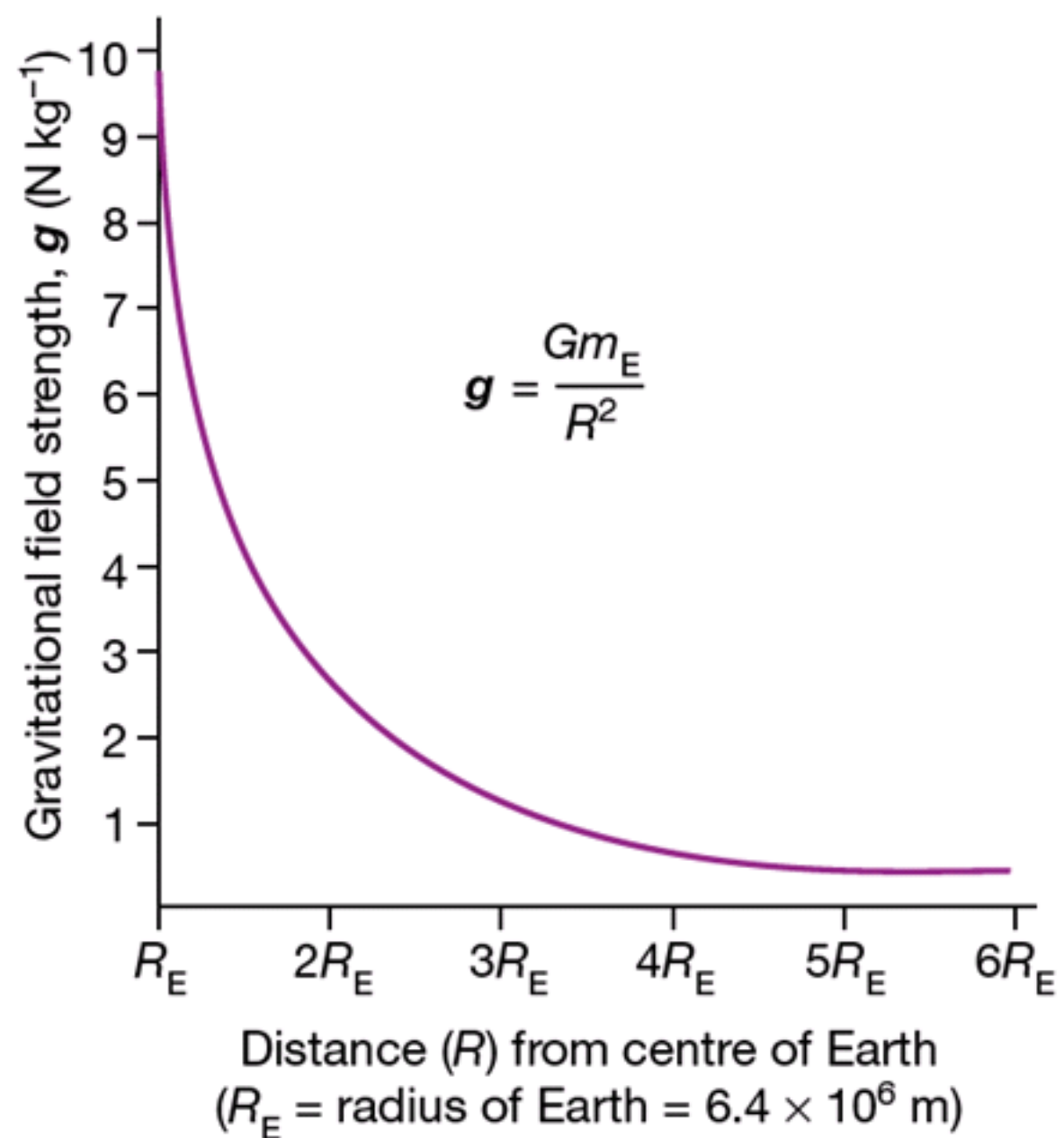


Figure 3.9 The gravitational field strength around the Earth. There is an inverse square relationship between the field strength, g , and the distance, R , from the centre of the Earth.

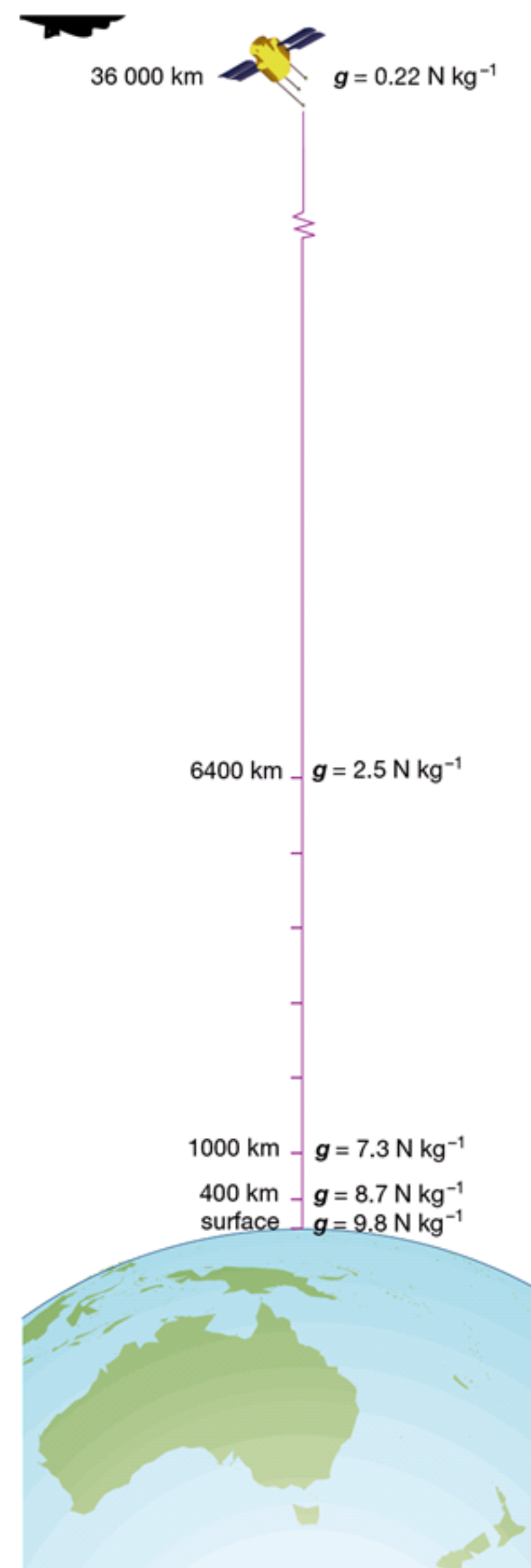
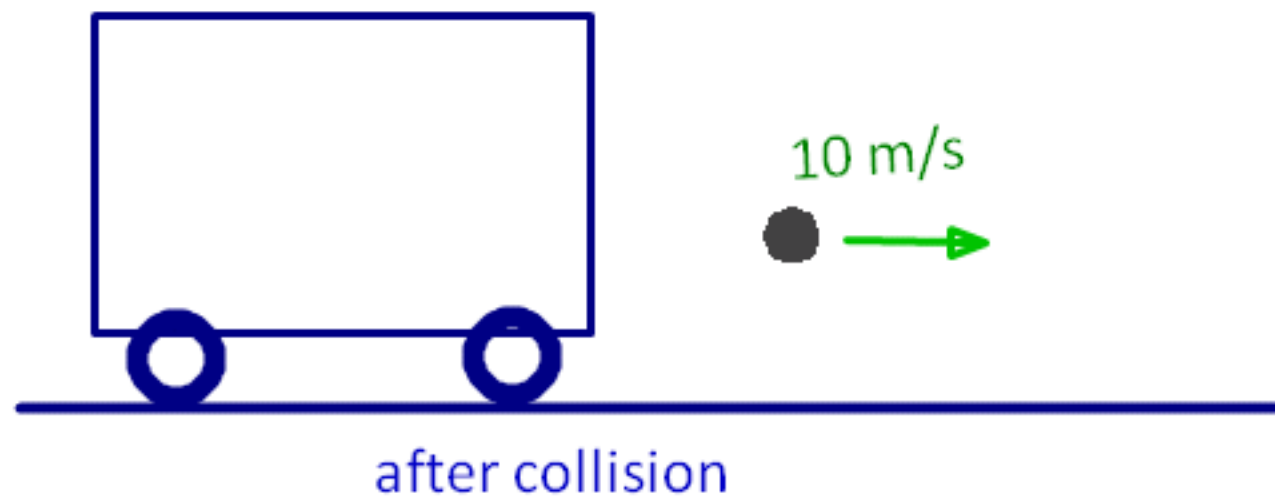
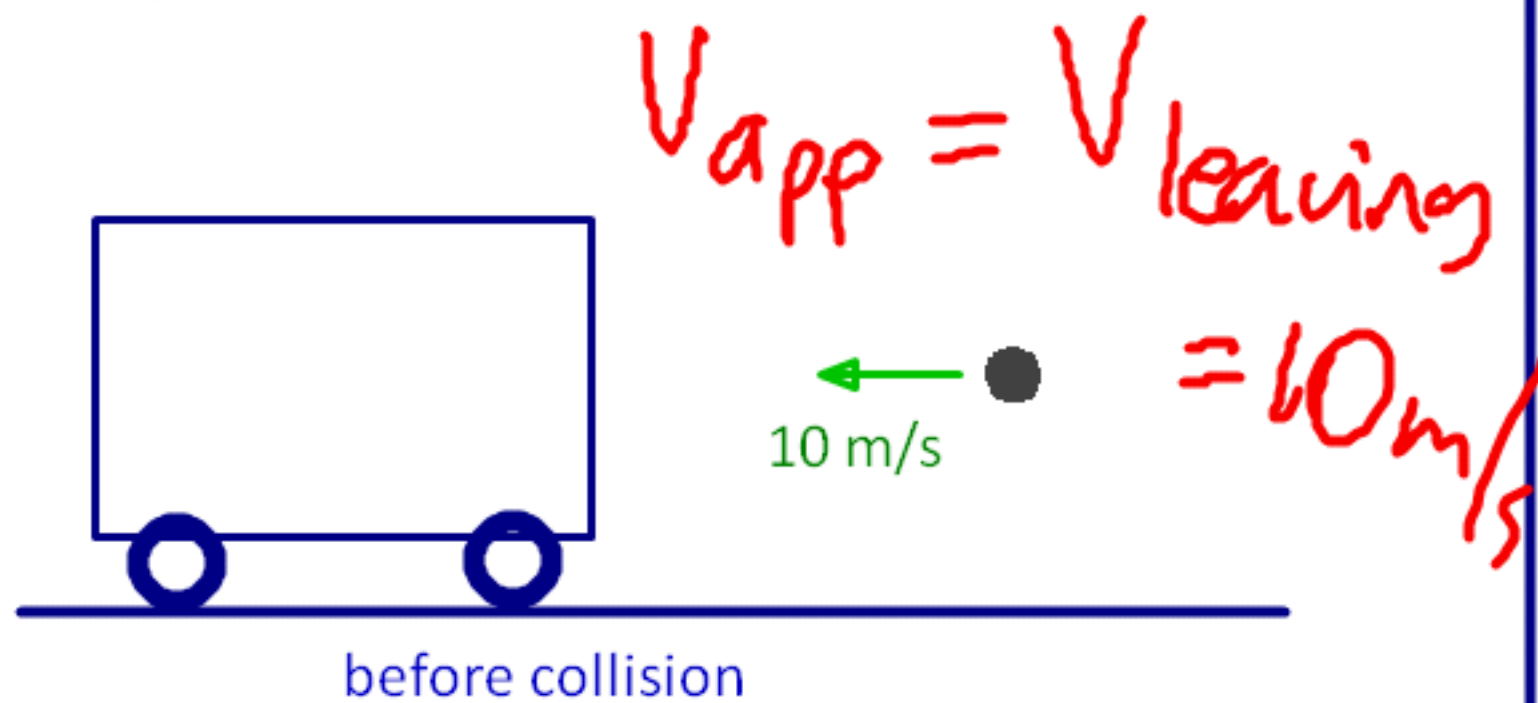


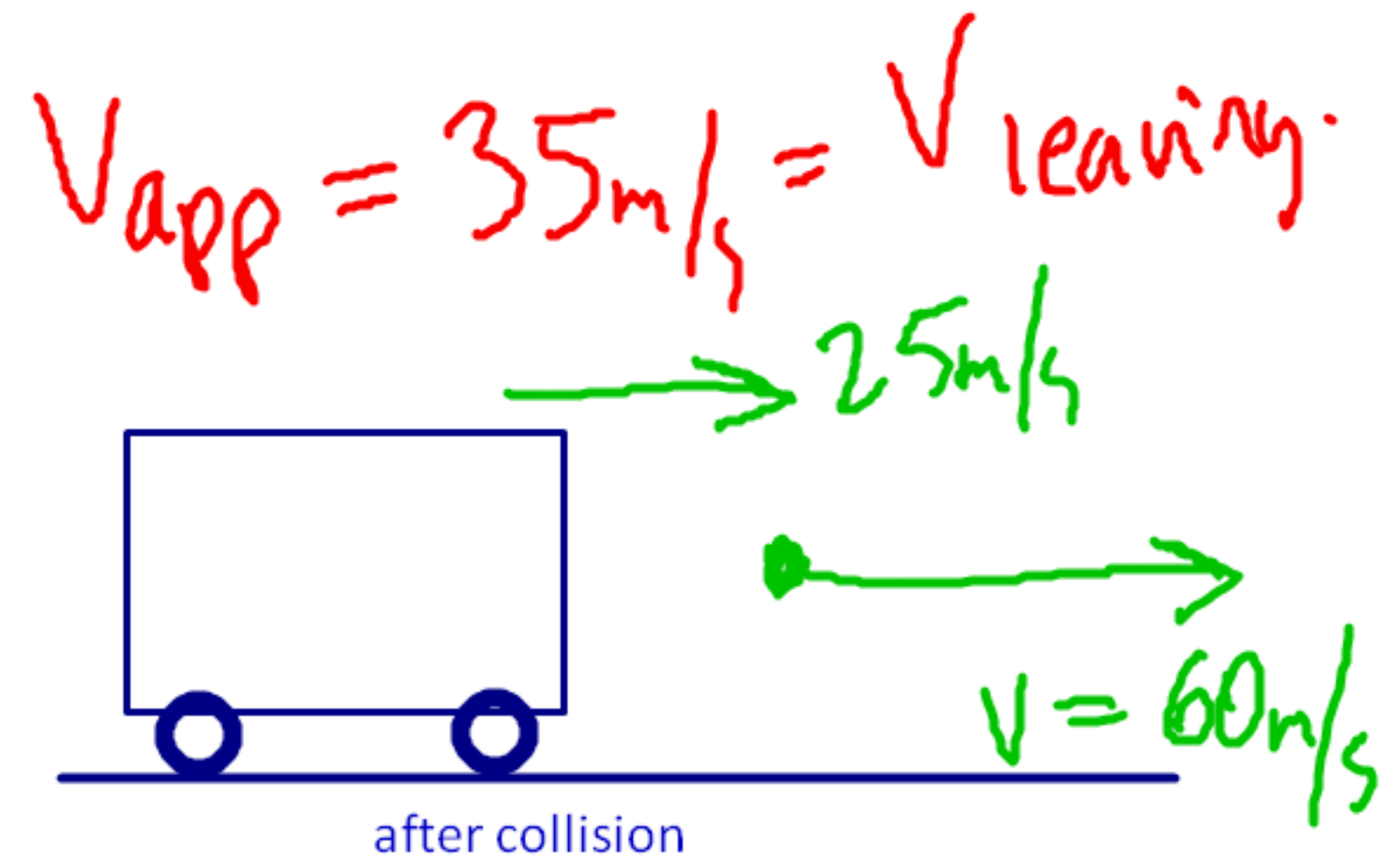
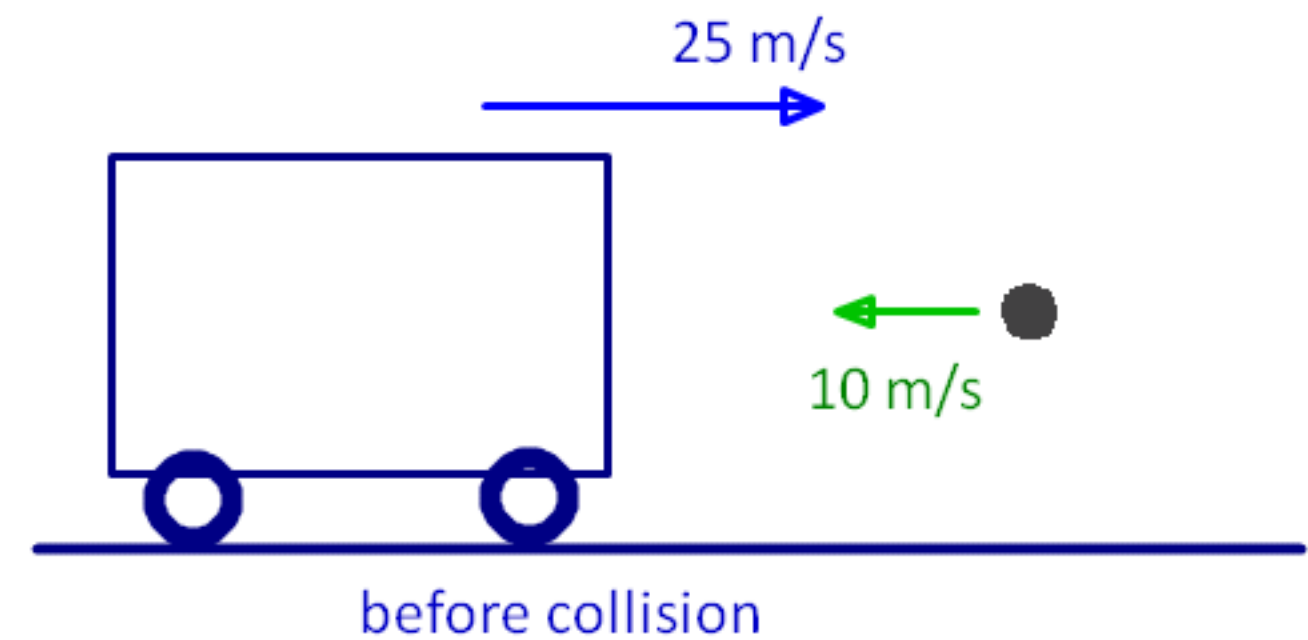
Figure 3.10 At 400 km above the Earth's surface, the gravitational field strength is 8.7 N kg^{-1} . The International Space Station has been in orbit at nearly this altitude since 1998. Australia's Optus communications satellites are in orbit at an altitude of about 36 000 km where Earth's gravitational field has a strength of only 0.22 N kg^{-1} .

WHAT IS THE "PHYSICS" BEHIND SLINGSHOT EFFECT?

Imagine a stationary train carriage is hit by a golf ball while travelling at 10 m/s left. What will be the final speed of the ball?



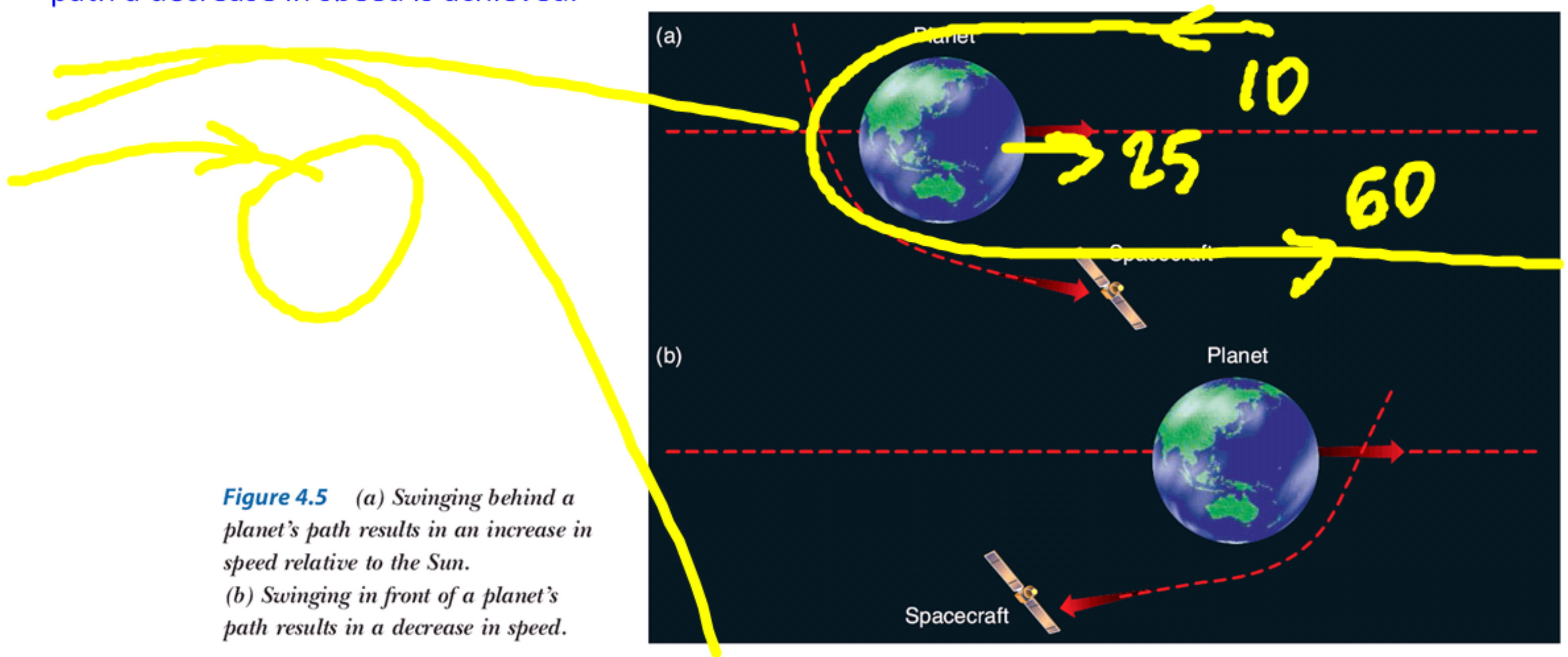
Imagine the same train now travelling at 25 m/s right hit by the same ball. What will be the final speed of the ball?



SLINGSHOT EFFECT (planetary swing-by or gravity-assist manoeuvre)

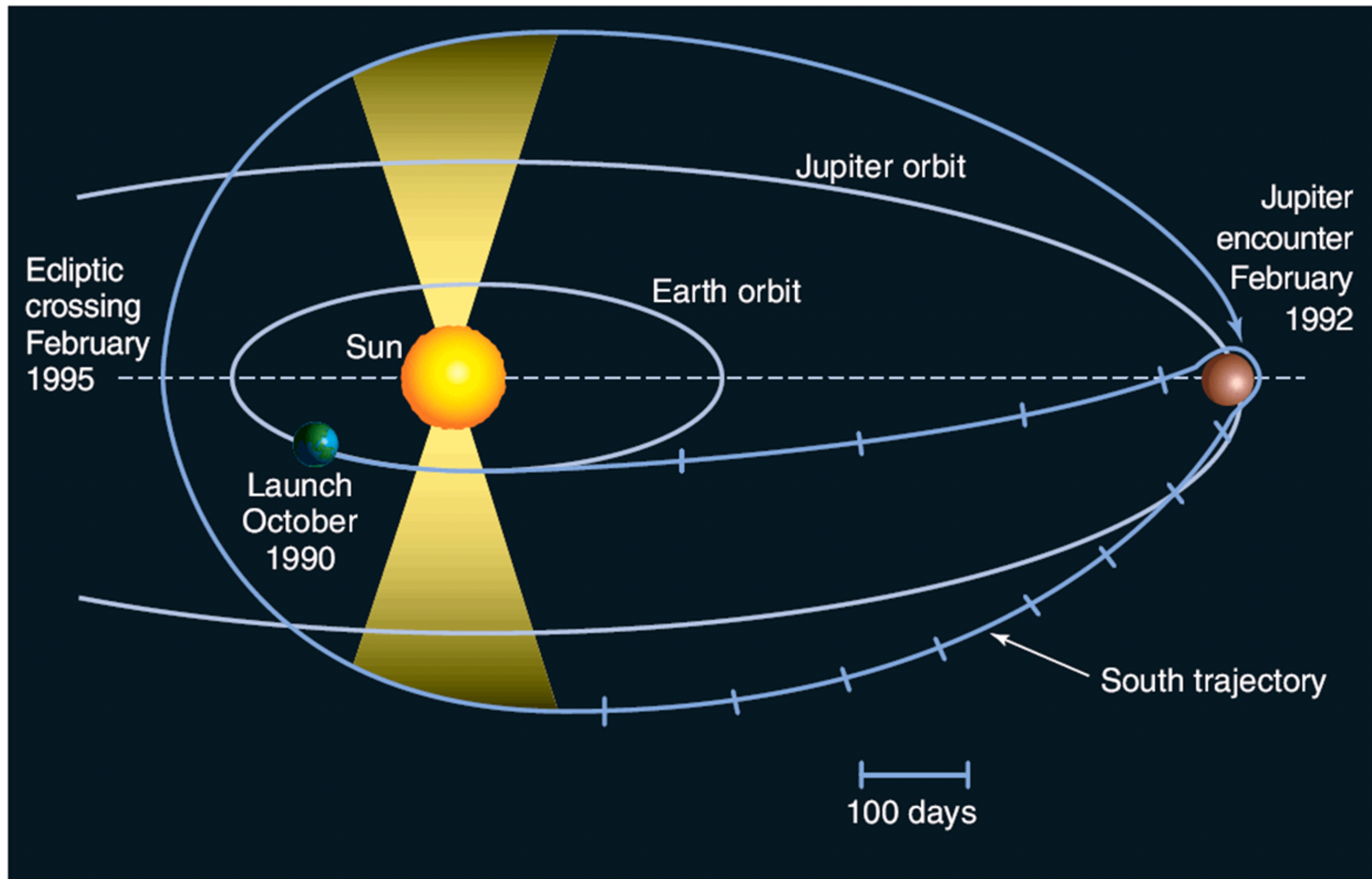
SLINGSHOT EFFECT (planetary swing-by or gravity-assist manoeuvre)

- ✓ A method used in space missions to pick up speed and proceed on to another target.
- ✓ During a slingshot, a spacecraft deliberately passes close to a large mass, such as a planet, so that the mass's gravity pulls the spacecraft in toward it.
- ✓ This causes the spacecraft to accelerate, and it heads around the planet and departs in a different direction.
- ✓ The departure speed of the spacecraft relative to the planet is the same as the approach speed relative to the planet, but the change in direction can result in a real change in velocity relative to the Sun.
- ✓ By swinging behind the planet an increase in speed can be achieved, and by swinging in front of the planet's path a decrease in speed is achieved.

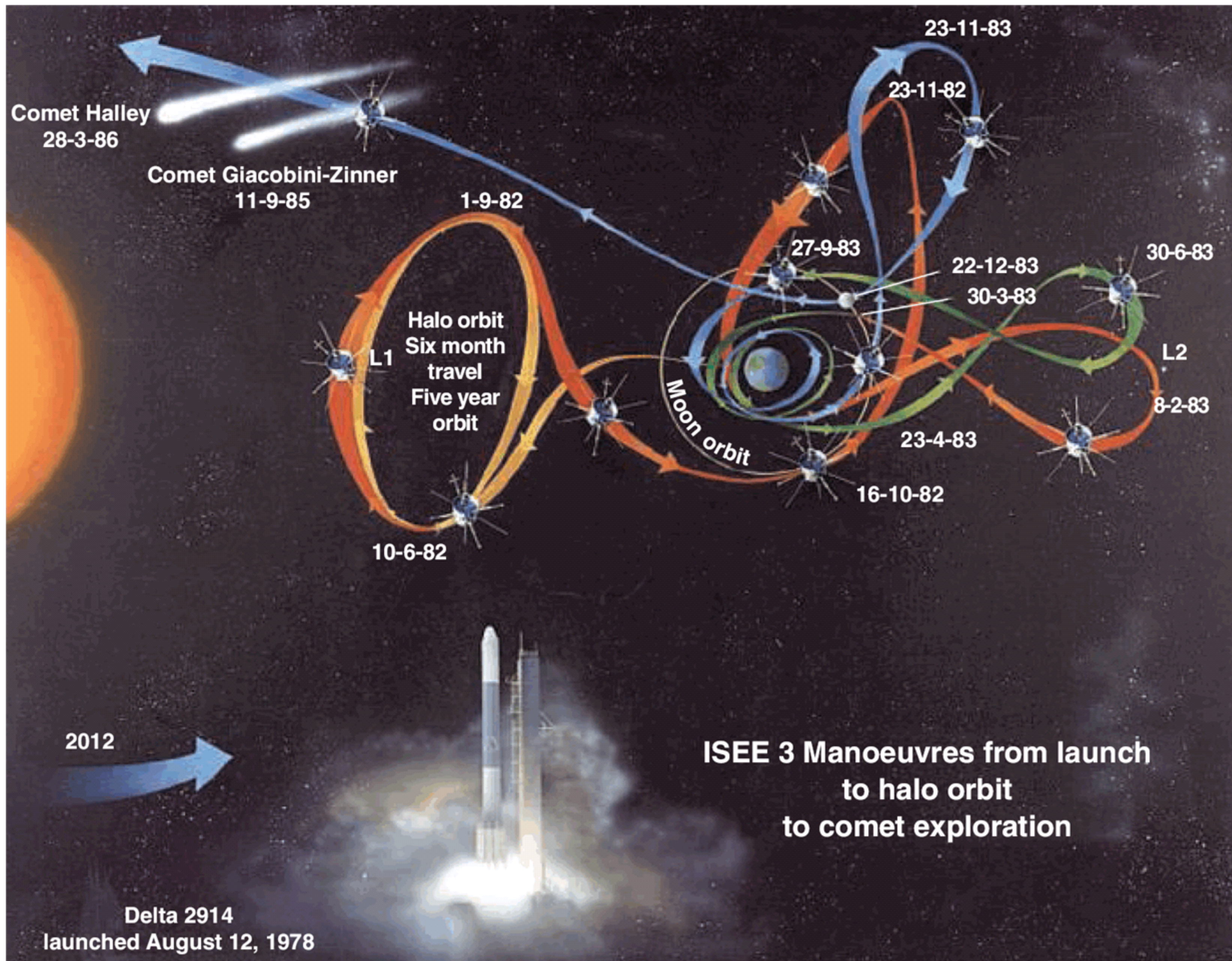


- ✓ The major benefit of the slingshot effect is that the change in velocity is achieved with very little expenditure of fuel.

SLINGSHOT OF ULYSSES AROUND JUPITER



TRAJECTORY OF ISEE-3/ICE



HOMEWORK

DUE

11 NOV

For these questions, assume that $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ and that the gravitational field strength on the surface of the Earth, g , is 9.8 N kg^{-1} .

The following information applies to questions 1–3.

The masses and radii of three planets are given in the following table.

Planet	Mass (kg)	Radius (m)
Mercury	3.30×10^{23}	2.44×10^6
Saturn	5.69×10^{26}	6.03×10^7
Jupiter	1.90×10^{27}	7.15×10^7

- 1 Calculate the gravitational field strength, g , at the surface of each planet.
- 2 Using your answers to Question 1, calculate the weight of an 80 kg astronaut on the surface of:
 - a Mercury
 - b Saturn
 - c Jupiter.
- 3 The result of your calculation for Question 1 should indicate that the gravitational field strength for Saturn is very close in value to that on Earth, i.e. approximately 10 N kg^{-1} . However, the Earth's radius and mass are very different from those of Saturn. How do you account for the fact that both planets have similar gravitational field strengths?

The following information applies to questions 4–6.

A 100 kg meteor is falling towards the Earth from a distance of 4.0 Earth radii from the centre of the Earth ($4.0R_E$).

- 4 Calculate g at this height.
- 5 What is the acceleration of the meteor at this height?
- 6 For this meteor determine the ratio:
$$\frac{\text{acceleration at } 4.0R_E}{\text{acceleration at } 2.0R_E}$$

The following information applies to questions 7–9.

There are bodies outside our Solar System, such as neutron stars, that produce very large gravitational fields. A typical neutron star can have a mass of $3.0 \times 10^{30} \text{ kg}$ and a radius of just 10 km.

- 7 Calculate the gravitational field strength at the surface of such a star.
- 8 Calculate the gravitational field strength at a distance of 5000 km from this star.
- 9 What would be the magnitude of the acceleration of a 10000-tonne asteroid located at this distance and falling towards this star?
- 10 A gravimeter is a device that can measure the Earth's gravitational field strength very accurately. Briefly explain how such a meter could be used to locate mineral deposits.
- 11 Two meteors, X and Y, are falling towards the Moon. Both are $3.0 \times 10^6 \text{ m}$ from the centre of the Moon. Meteor X has a mass of 500 kg and meteor Y has a mass of 50 kg. The mass of the Moon is $7.3 \times 10^{22} \text{ kg}$. Calculate the:
 - a gravitational force acting on X
 - b acceleration of X
 - c gravitational force acting on Y
 - d acceleration of Y
 - e gravitational field strength at this location.
- 12 There is a point between the Earth and the Moon where the total gravitational field is zero. The significance of this is that returning lunar missions are able to return to Earth under the influence of the Earth's field once they pass this point. Given that the mass of Earth is $6.0 \times 10^{24} \text{ kg}$, the mass of the Moon is $7.3 \times 10^{22} \text{ kg}$ and the radius of the Moon's orbit is $3.8 \times 10^8 \text{ m}$, calculate the distance of this point from the centre of the Earth.

HOMEWORK

- ✦ Homework is an integral part of your "Learning Curve", take it seriously!
- ✦ Target minimum 1 hour of Physics everyday
- ✦ Divide your physics home study in three segments;
 - ✓ Revision (past)
 - ✓ Homework (present)
 - ✓ Tomorrow (future)
- ✦ Homework is due next period, unless otherwise stated
- ✦ If you cannot do all, at least do a few from each piece

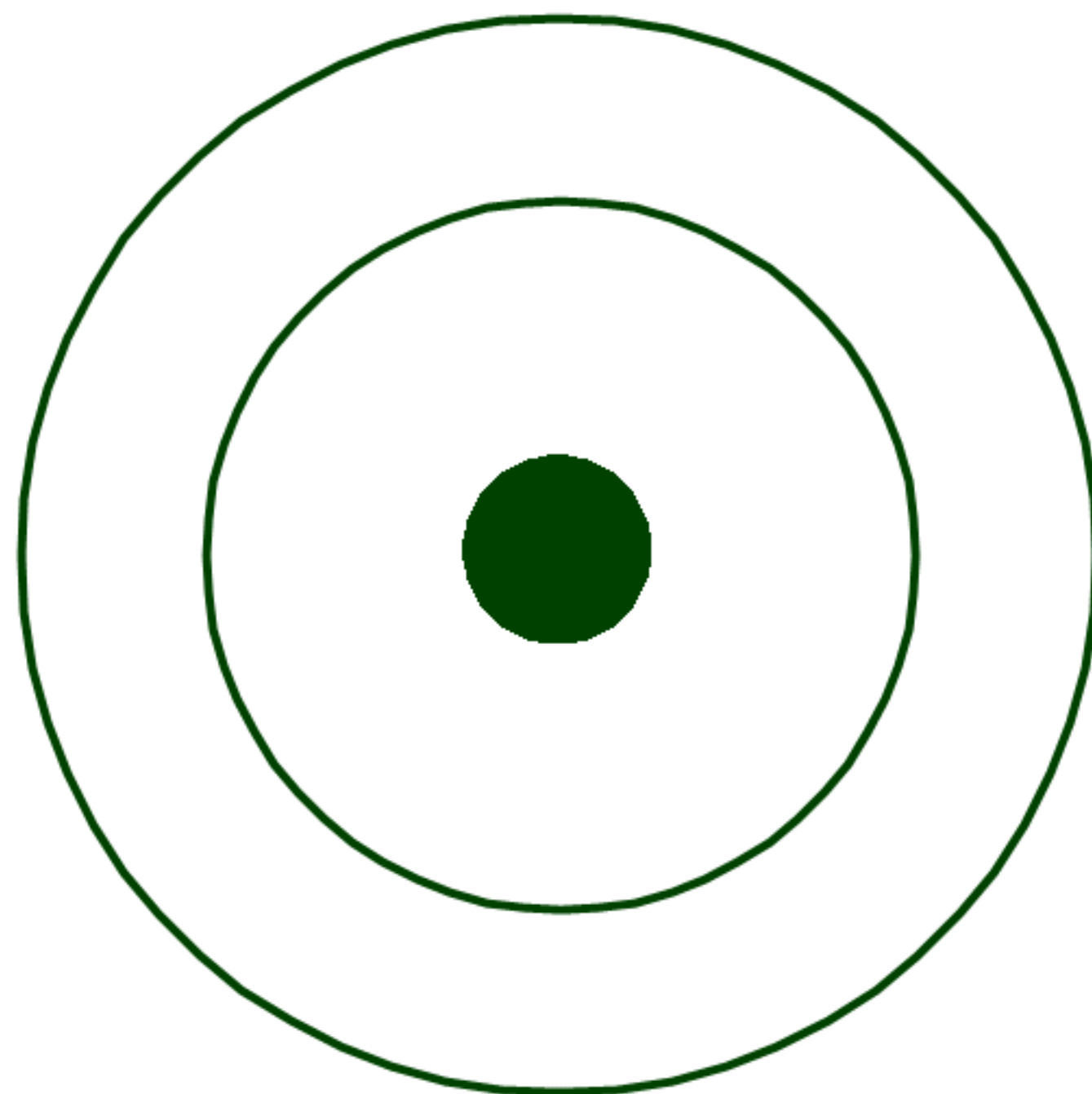
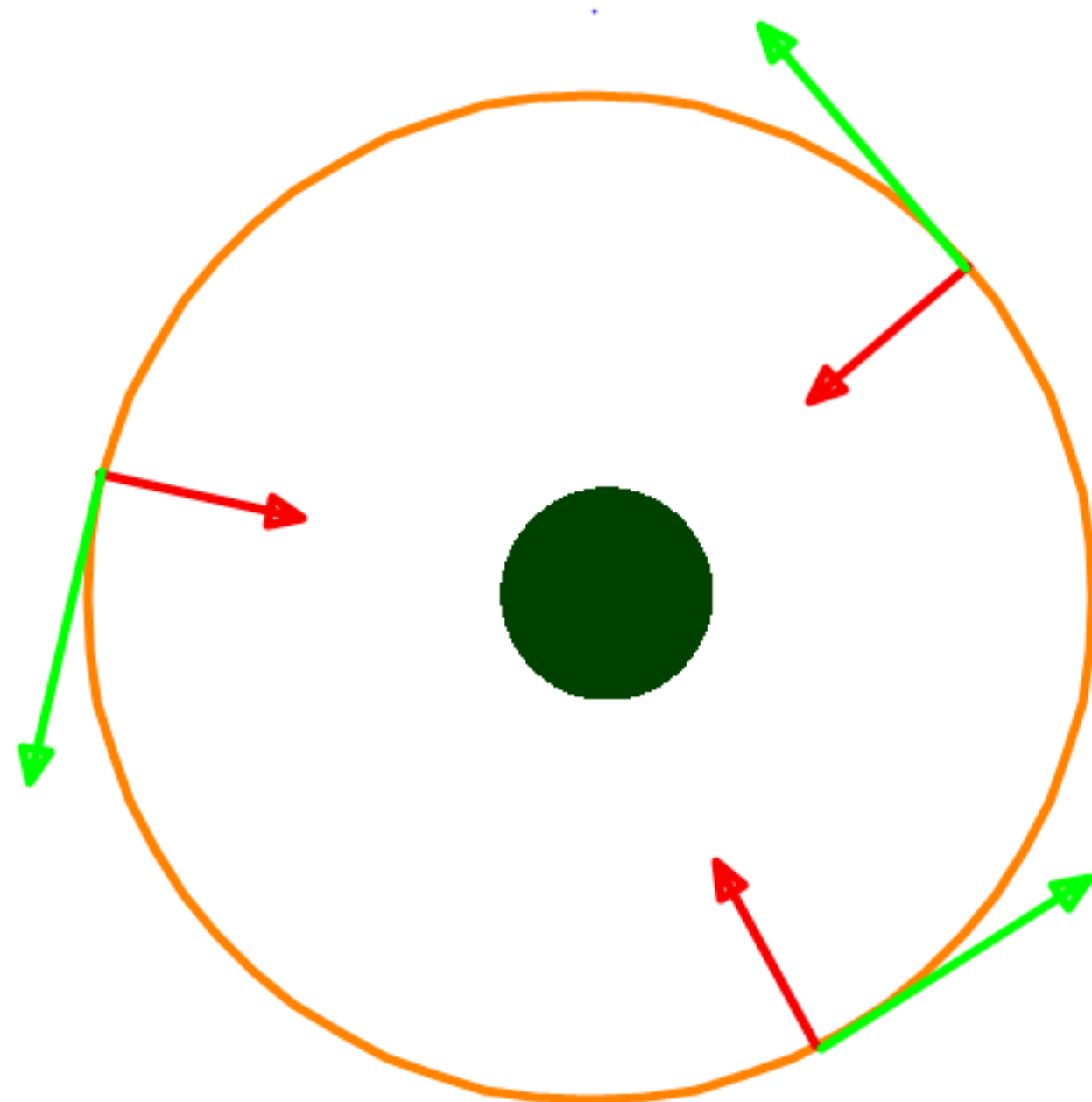
*Apart from **reading the relevant pages from the textbook and solving the rest of the questions in this booklet** your homework is:*

- ✓ 12 questions in this booklet
- ✓ Chapter 4 all questions
- ✓ 8 questions of P17
- ✓ Experiment 5 Report
- ✓ Relevant pages in Multiple Choice Dot Points Book (DPB)
- ✓ New 8 page booklet (pages 16-23)
- ✓ Chapter 3 all questions

NEXT PERIOD > SLINGSHOT EFFECT

COMPARISON OF COMMON CIRCULAR MOTIONS

MOTION	F_c PROVIDED BY ...
Whirling rock on a string	The string
Electron orbiting atomic nucleus	Electron–nucleus electrical attraction
Car cornering	Friction between tyres and road
Moon revolving around Earth	Moon–Earth gravitational attraction
Satellite revolving around Earth	Satellite–Earth gravitational attraction
Lady bug on a rotating disc	Friction b/w her hairy feet and the disc
Principal rotating the girl in the movie "Mathilda"	Tension in their arms
Playground swing	Tension in the rope
Vortex	Reaction force from the walls
Earth revolving around the Sun	Earth-Sun gravitational attraction



Steps in solving PM questions.

Step 1 > Read the question.

Step 2 > Understand the question.

Step 3 > Make sure you understand "What is given/provided" and "What is asked".

Step 4 > Draw a diagram.

Step 5 > Select your interval (A to B). Mark A and B on your diagram.

Step 6 > Draw the data table and fill in the details as much as you can. Mark unknowns.

Step 7 > Select the appropriate formula and solve it for unknowns.