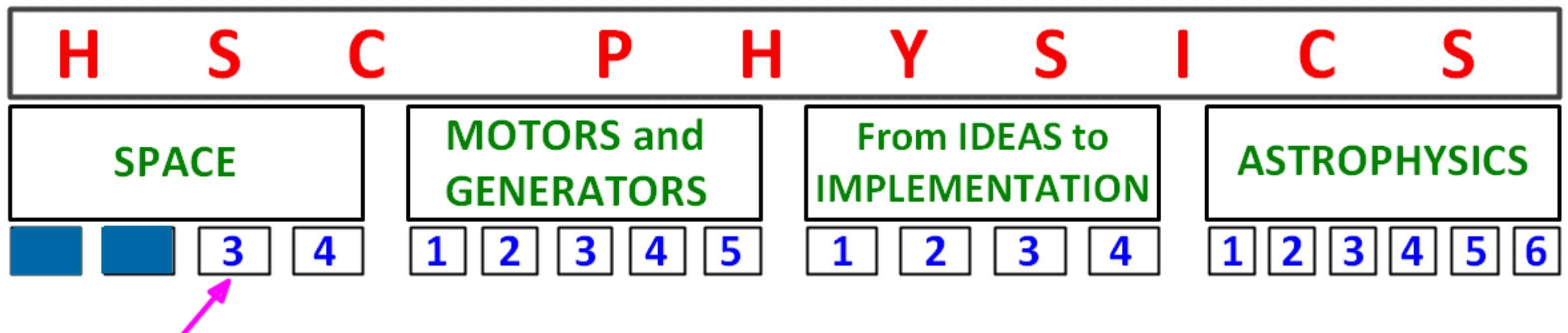


SPACE

1st Quarter; Module 1

PERIOD 19

Gravitational Field and Slingshot Effect



REVISION OF FORMULAS.

$$E_p = -\frac{GMm}{r}$$

$$F_c = \frac{mv^2}{r}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$v_c = \frac{2\pi r}{T} \text{ (orbital velocity)}$$

$$F_g = G \frac{mM}{d^2}$$

$$F_g = F_c$$

$$G \frac{mM}{d^2} = \frac{mv^2}{d}$$

$$v_{orb} = \sqrt{\frac{GM}{d}}$$

$$v_{orb} = \frac{2\pi r}{T}$$

$$F_g = F_{net}$$

$$G \frac{mM}{d^2} = m \cdot a$$

$$a = g = \frac{GM}{d^2}$$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$\frac{1}{2}mv^2 + -\frac{GMm}{r} = 0$$

$$E_k + E_p = 0 \text{ at space}$$

$$F_c = F_{net}$$

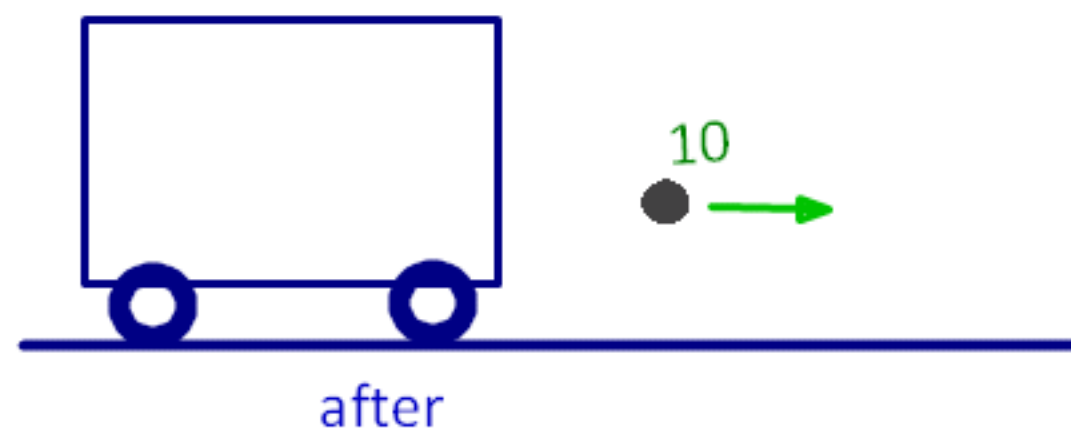
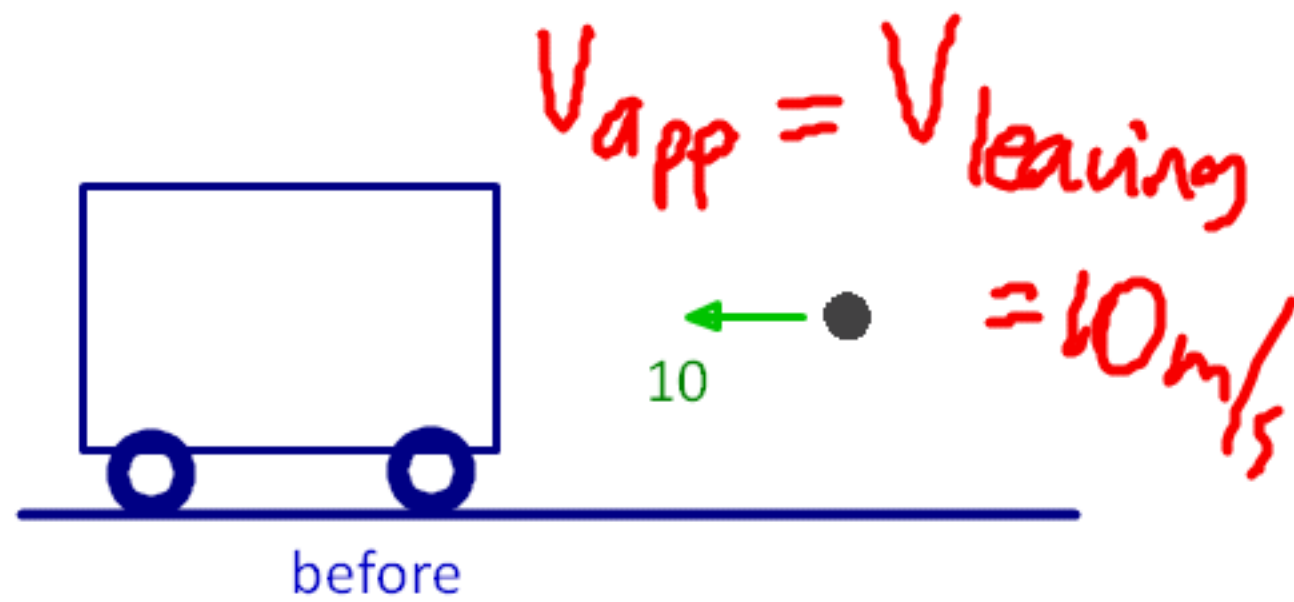
$$\frac{mv^2}{r} = m \cdot a$$

$$a = \frac{v^2}{r}$$

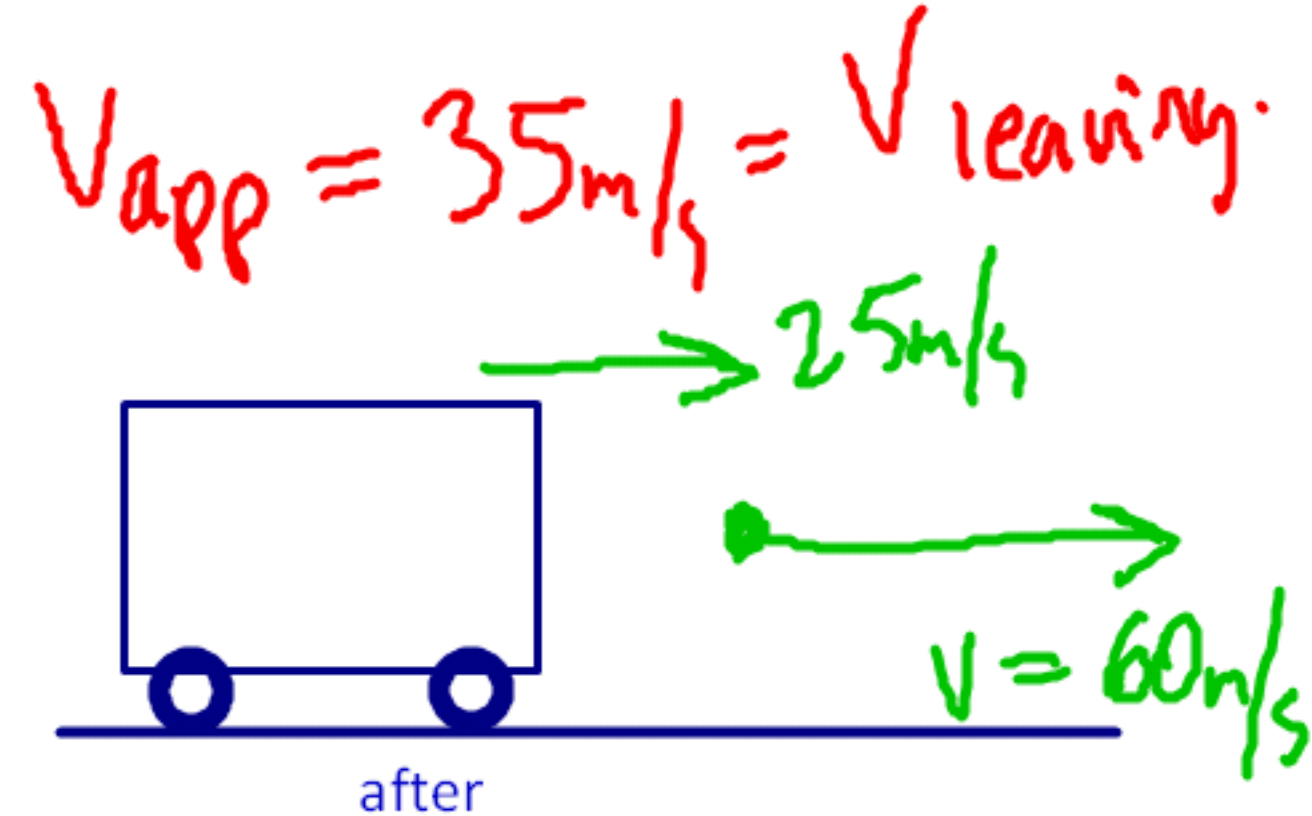
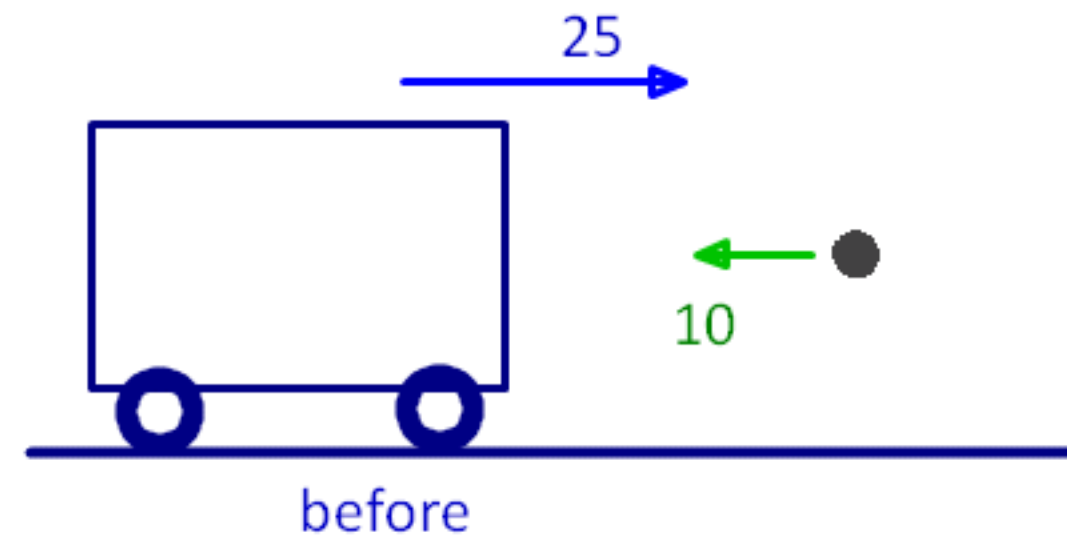
equal

WHAT IS THE "PHYSICS" BEHIND SLINGSHOT EFFECT?

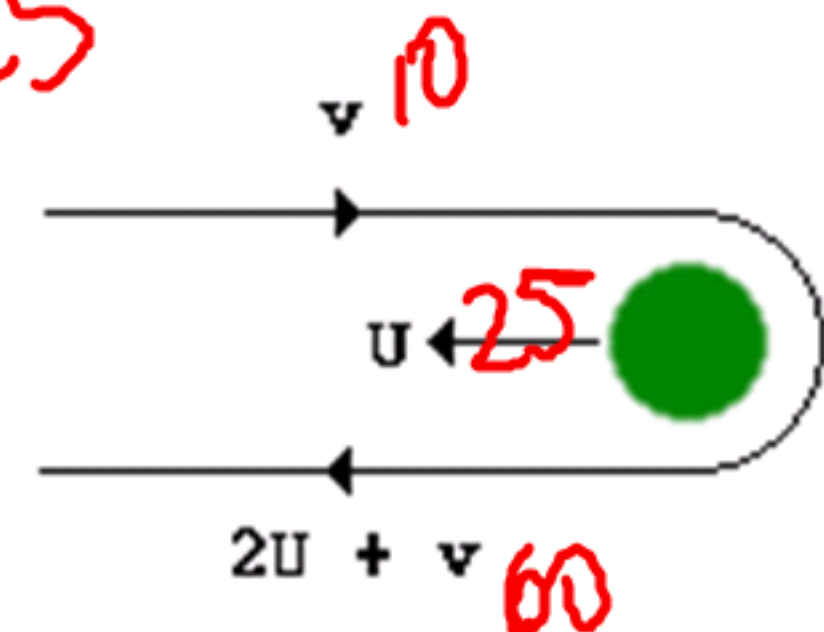
Imagine a stationary train carriage is hit by a golf ball while travelling at 10 m/s left.
What will be the final speed of the ball?



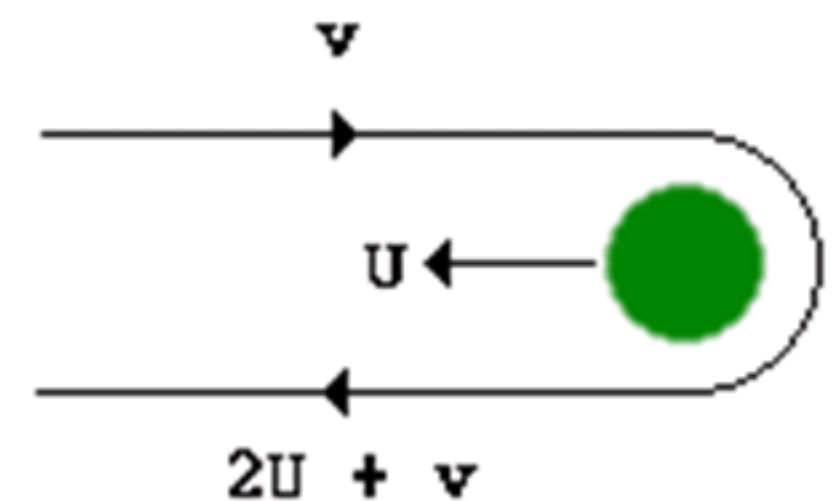
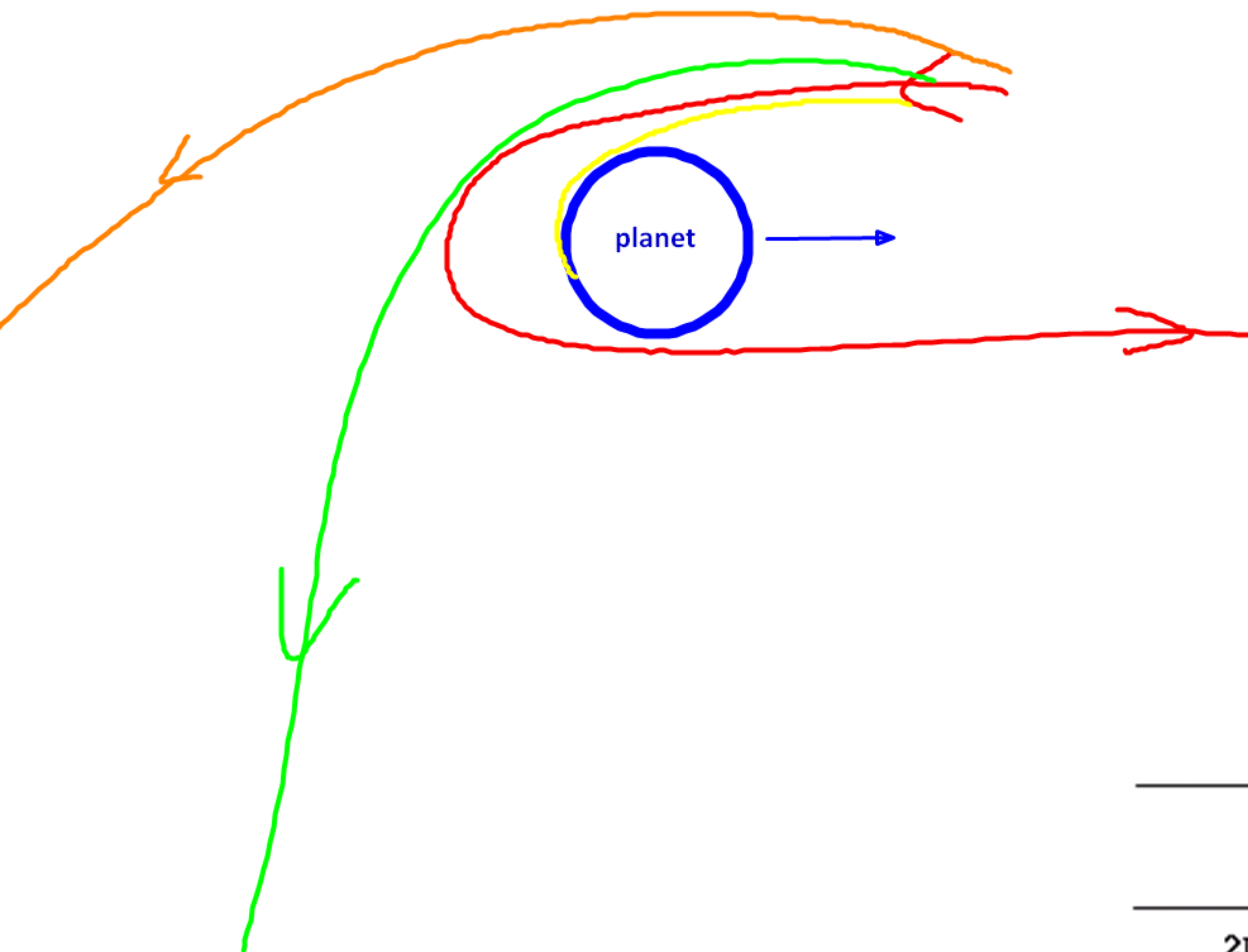
Imagine the same train now travelling at 25 m/s right hit by the same ball. What will be the final speed of the ball?



$$60 = 10 \times 2.25$$



SLINGSHOT EFFECT (planetary swing-by or gravity-assist manoeuvre)



SLINGSHOT EFFECT (planetary swing-by or gravity-assist manoeuvre)

HOMEWORK

- ✦ Homework is an integral part of your "Learning Curve", take it seriously!
- ✦ Target minimum 1 hour of Physics everyday
- ✦ Divide your physics home study in three segments;
 - ✓ Revision (past)
 - ✓ Homework (present)
 - ✓ Tomorrow (future)
- ✦ Homework is due next period, unless otherwise stated
- ✦ If you cannot do all, at least do a few from each piece

*Apart from **reading the relevant pages from the textbook and solving the rest of the questions in this booklet***

your homework is:

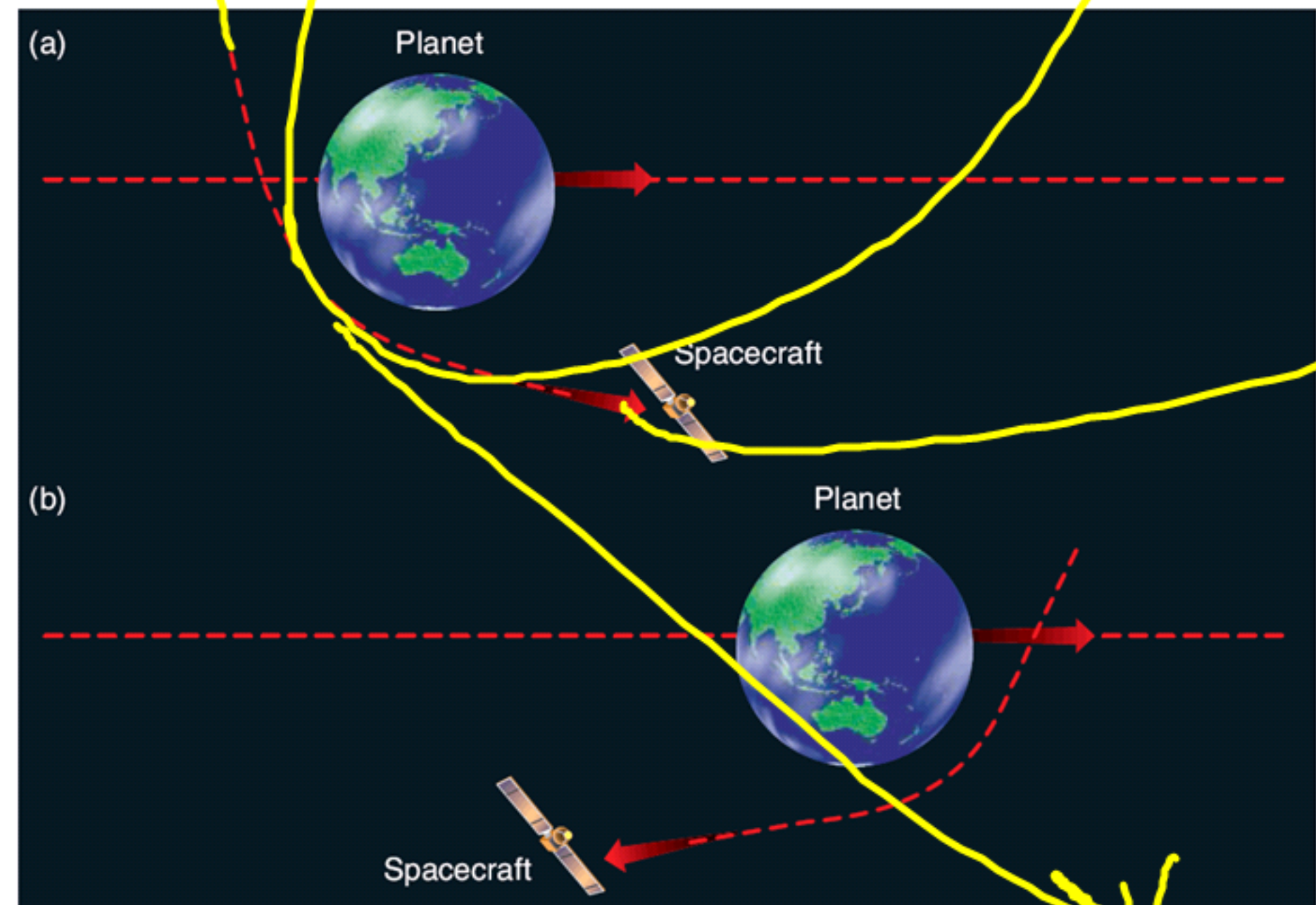
- ✓ Study CSU Space 3 notes
- ✓ New Dot Points booklet (pages 24-27)
- ✓ 12 questions in this booklet
- ✓ Chapter 4 all questions
- ✓ 12 questions of P18
- ✓ 8 questions of P17
- ✓ Experiment 5 Report
- ✓ Relevant pages in Multiple Choice Dot Points Book (DPB)
- ✓ New 8 page booklet (pages 16-23)
- ✓ Chapter 3 all questions

NEXT PERIOD > EITHER THEORY - MICHELSON&MORLEY EXPERIMENT

SLINGSHOT EFFECT (planetary swing-by or gravity-assist manoeuvre)

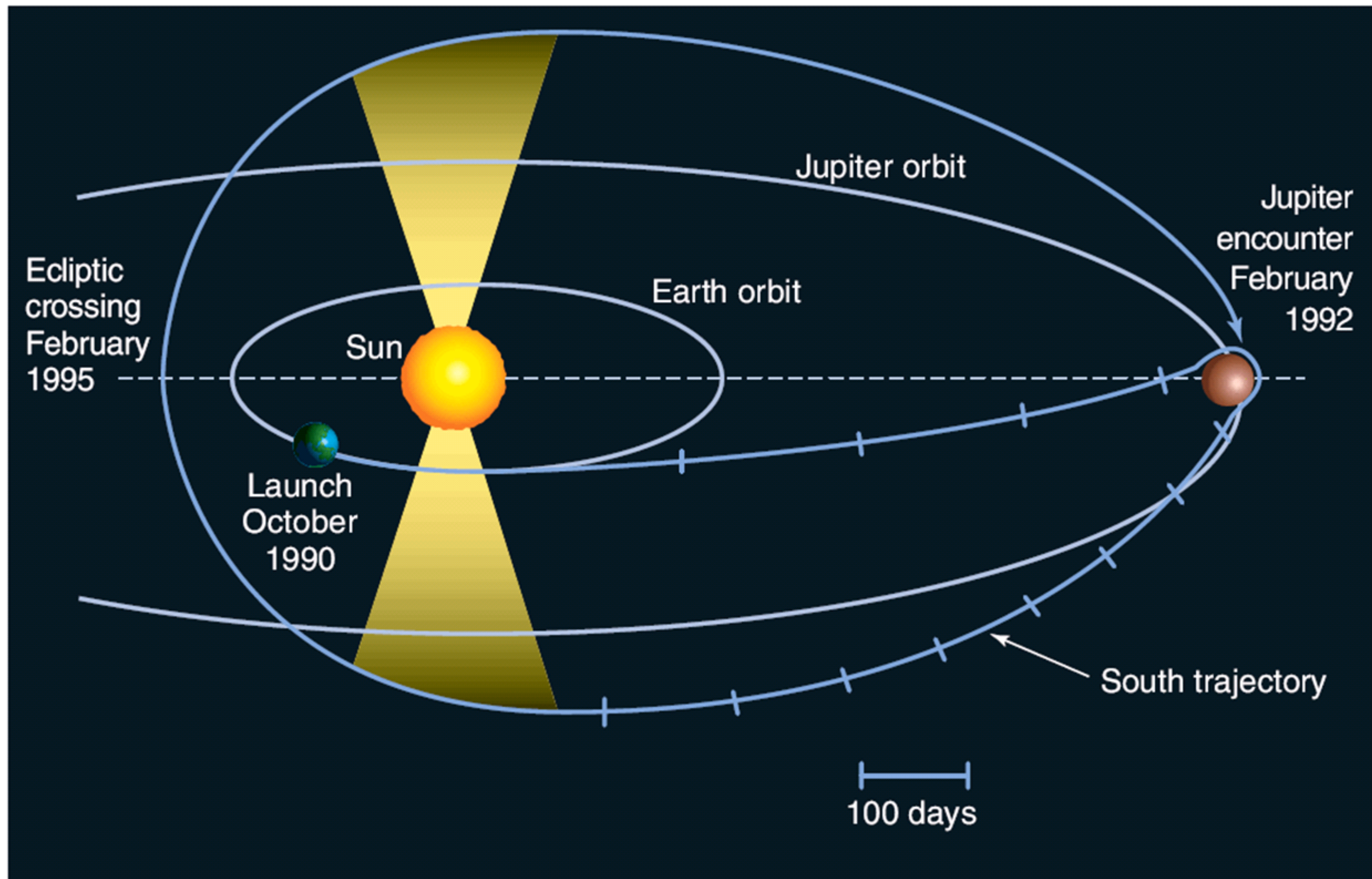
- ✓ A method used in space missions to pick up speed and proceed on to another target.
- ✓ During a slingshot, a spacecraft deliberately passes close to a large mass, such as a planet, so that the mass's gravity pulls the spacecraft in toward it.
- ✓ This causes the spacecraft to accelerate, and it heads around the planet and departs in a different direction.
- ✓ The departure speed of the spacecraft relative to the planet is the same as the approach speed relative to the planet, but the change in direction can result in a real change in velocity relative to the Sun.
- ✓ By swinging behind the planet an increase in speed can be achieved, and by swinging in front of the planet's path a decrease in speed is achieved.

Figure 4.5 (a) *Swinging behind a planet's path results in an increase in speed relative to the Sun.*
(b) *Swinging in front of a planet's path results in a decrease in speed.*

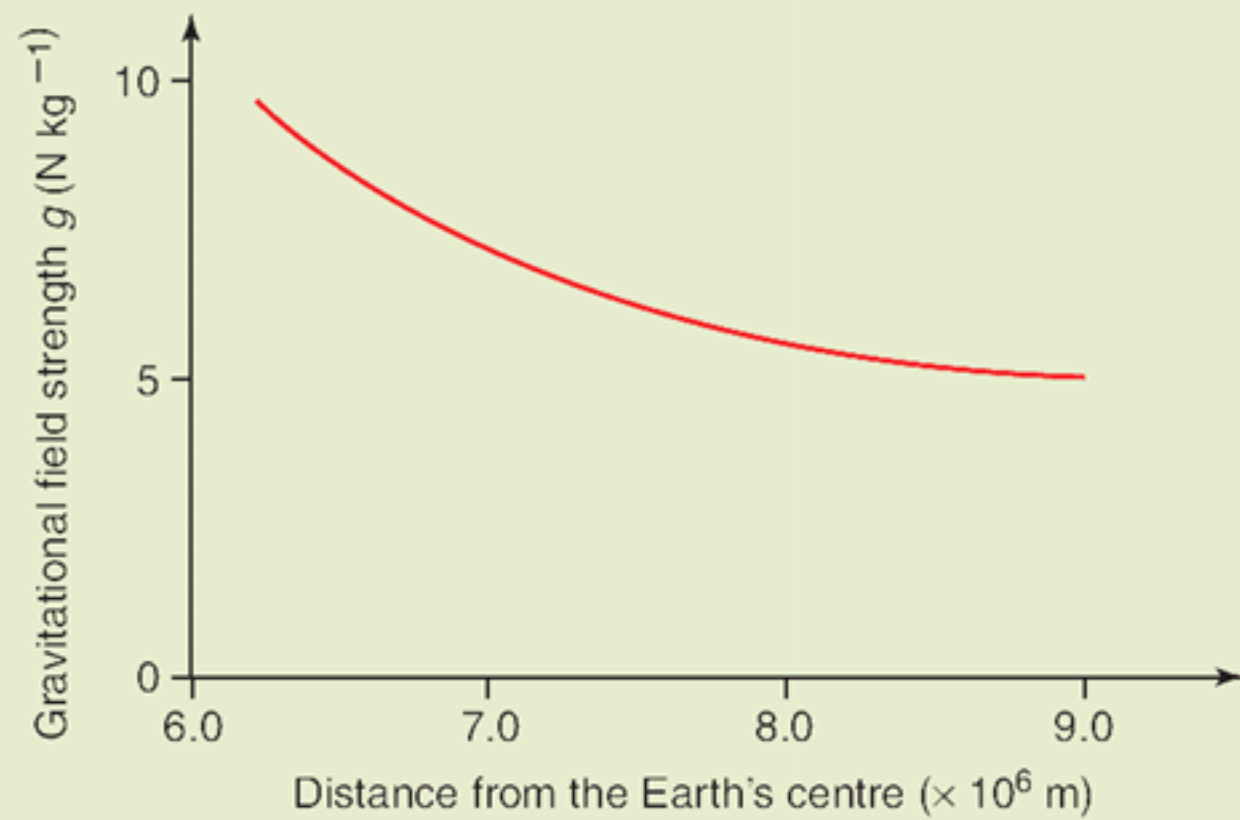


- ✓ The major benefit of the slingshot effect is that the change in velocity is achieved with very little expenditure of fuel.

SLINGSHOT OF ULYSSES AROUND JUPITER



48. A disabled satellite of mass 2400 kg is in orbit around Earth at a height of 2000 km above sea level. It falls to a height of 800 km before its built-in rocket system can be activated to stop the fall continuing.



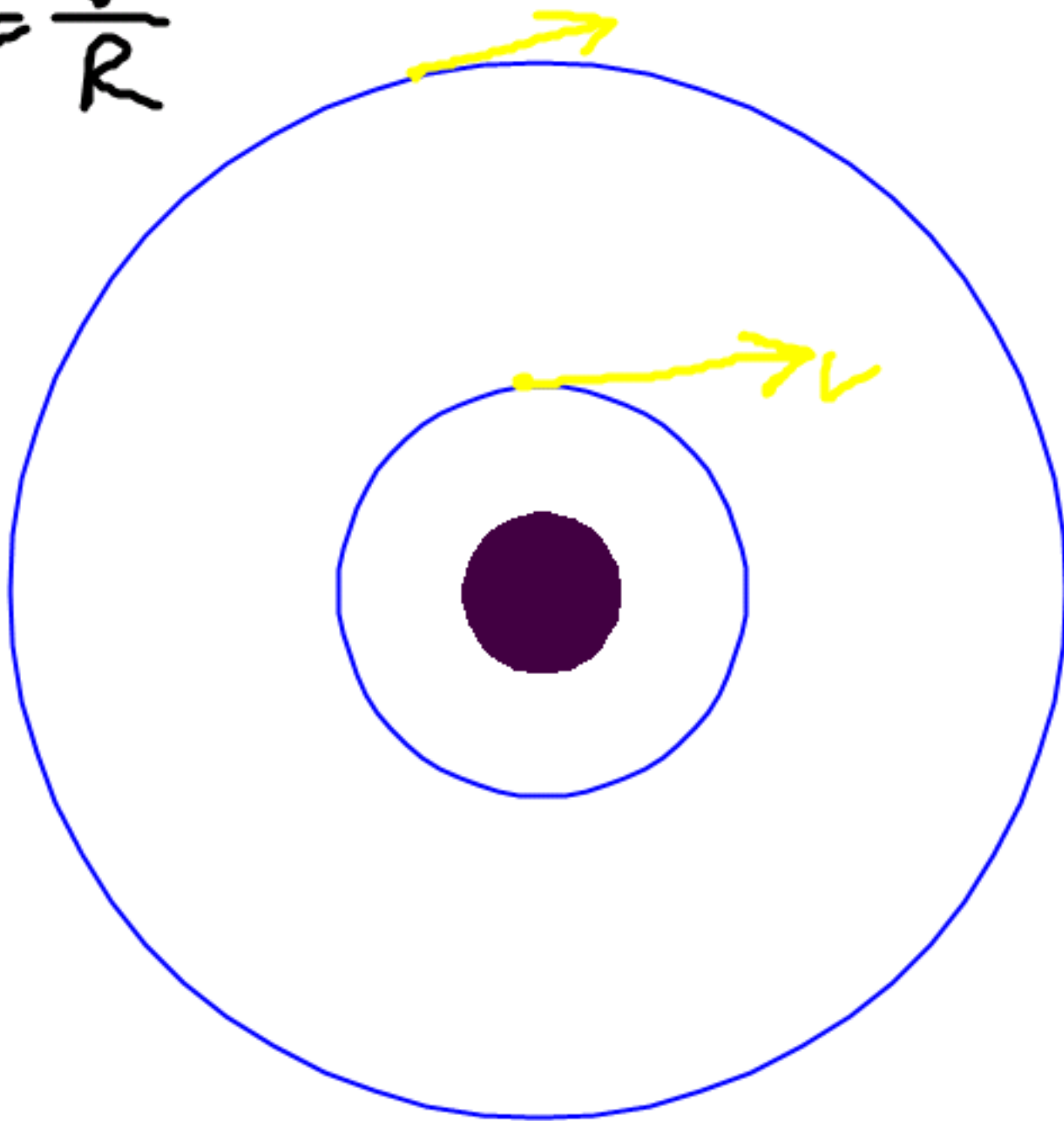
- Calculate the gravitational force on the satellite while it is in its initial orbit.
- Calculate the loss of gravitational potential energy of the satellite during its fall.
- If the speed of the satellite during its initial orbit is 6900 m s^{-1} , what is its speed when the rocket system is activated?

- 9 The data for two of Saturn's moons, Atlas and Helene, is as follows. The orbit of Helene is about twice as far from the centre of Saturn as that of Atlas.

	Orbital radius (m)	Orbital period (days)
Atlas	1.37×10^8	0.602
Helene	3.77×10^8	2.75

- a Calculate the value of these ratios:
- i orbital speed of Atlas/orbital speed of Helene
 - ii acceleration of Atlas/acceleration of Helene.
- b The largest of Saturn's moons is Titan. It has an orbital radius of 1.20×10^9 m. Use Kepler's third law to show that the orbital period of Titan is 15.6 days.

$$a = \frac{v^2}{R}$$



i)
$$\frac{v_A}{v_H} = \frac{\frac{2\pi R_A}{T_A}}{\frac{2\pi R_H}{T_H}} = \frac{1.37}{0.602} \div \frac{3.77}{2.75} = 1.66$$

ii)
$$\frac{a_A}{a_H} = \frac{\frac{GM}{R_A^2}}{\frac{GM}{R_H^2}} = \frac{v_A^2/R_A}{v_H^2/R_H}$$

Find mass of Saturn!

for Atlas
Kepler's 3rd Law

$$\frac{R_A^3}{T_A^2} = \frac{G \cdot M_S}{4\pi^2}$$

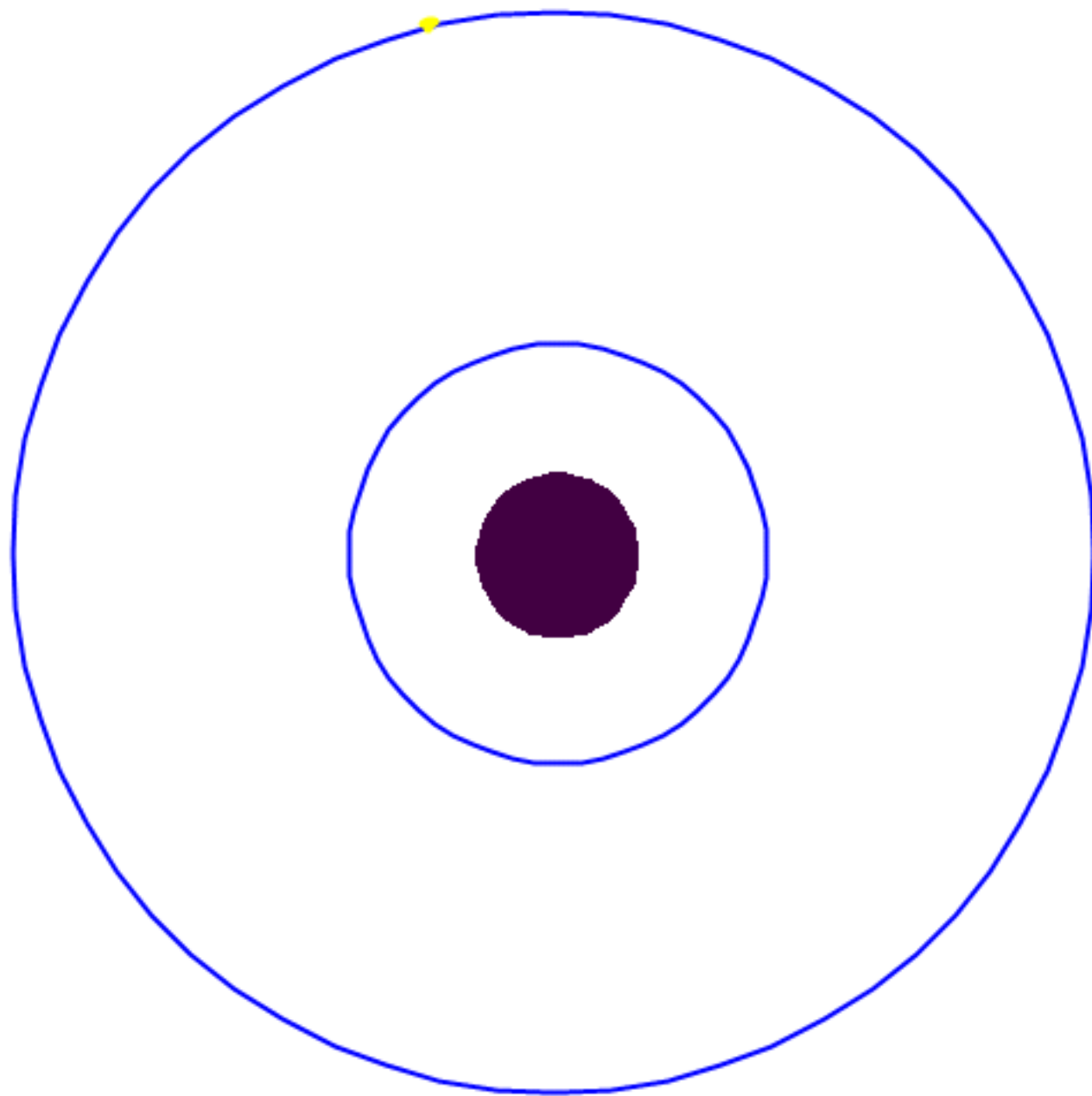
$$M_S = \frac{R_A^3 \cdot 4\pi^2}{T_A^2 \cdot G}$$

$$= 5.6 \times 10^{26} \text{ kg} \quad \checkmark$$

- 9 The data for two of Saturn's moons, Atlas and Helene, is as follows. The orbit of Helene is about twice as far from the centre of Saturn as that of Atlas.

	Orbital radius (m)	Orbital period (days)
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- b The largest of Saturn's moons is Titan. It has an orbital radius of 1.20×10^9 m. Use Kepler's third law to show that the orbital period of Titan is 15.6 days.



$$\frac{R_A^3}{T_A^2} = \frac{R_T^3}{T_T^2} \quad \text{(days)}$$

$$\frac{(1.37 \times 10^8)^3}{0.602^2} = \frac{(1.20 \times 10^9)^3}{T_T^2}$$

$$T_T = 15.6 \text{ days}$$

12 There is a point between the Earth and the Moon where the total gravitational field is zero. The significance of this is that returning lunar missions are able to return to Earth under the influence of the Earth's field once they pass this point. Given that the mass of Earth is 6.0×10^{24} kg, the mass of the Moon is 7.3×10^{22} kg and the radius of the Moon's orbit is 3.8×10^8 m, calculate the distance of this point from the centre of the Earth.

$$g_E = g_m$$

$$d = 380000 \text{ km} = 3.8 \times 10^8 \text{ m}$$



$$\frac{\cancel{G} \cdot M_E}{x^2} = \frac{\cancel{G} \cdot M_m}{(3.8 \times 10^8 - x)^2}$$

$$g_E = g_m$$

PERIOD 18

QUESTIONS

For these questions, assume that $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ and that the gravitational field strength on the surface of the Earth, g , is 9.8 N kg^{-1} .

The following information applies to questions 1–3.

The masses and radii of three planets are given in the following table.

Planet	Mass (kg)	Radius (m)
Mercury	3.30×10^{23}	2.44×10^6
Saturn	5.69×10^{26}	6.03×10^7
Jupiter	1.90×10^{27}	7.15×10^7

- 1 Calculate the gravitational field strength, g , at the surface of each planet.
- 2 Using your answers to Question 1, calculate the weight of an 80 kg astronaut on the surface of:
 - a Mercury
 - b Saturn
 - c Jupiter.
- 3 The result of your calculation for Question 1 should indicate that the gravitational field strength for Saturn is very close in value to that on Earth, i.e. approximately 10 N kg^{-1} . However, the Earth's radius and mass are very different from those of Saturn. How do you account for the fact that both planets have similar gravitational field strengths?

The following information applies to questions 4–6.

A 100 kg meteor is falling towards the Earth from a distance of 4.0 Earth radii from the centre of the Earth ($4.0R_E$).

- 4 Calculate g at this height.
- 5 What is the acceleration of the meteor at this height?
- 6 For this meteor determine the ratio:
$$\frac{\text{acceleration at } 4.0R_E}{\text{acceleration at } 2.0R_E}$$

The following information applies to questions 7–9.

There are bodies outside our Solar System, such as neutron stars, that produce very large gravitational fields. A typical neutron star can have a mass of $3.0 \times 10^{30} \text{ kg}$ and a radius of just 10 km.

- 7 Calculate the gravitational field strength at the surface of such a star.
- 8 Calculate the gravitational field strength at a distance of 5000 km from this star.
- 9 What would be the magnitude of the acceleration of a 10000-tonne asteroid located at this distance and falling towards this star?
- 10 A gravimeter is a device that can measure the Earth's gravitational field strength very accurately. Briefly explain how such a meter could be used to locate mineral deposits.
- 11 Two meteors, X and Y, are falling towards the Moon. Both are $3.0 \times 10^6 \text{ m}$ from the centre of the Moon. Meteor X has a mass of 500 kg and meteor Y has a mass of 50 kg. The mass of the Moon is $7.3 \times 10^{22} \text{ kg}$. Calculate the:
 - a gravitational force acting on X
 - b acceleration of X
 - c gravitational force acting on Y
 - d acceleration of Y
 - e gravitational field strength at this location.
- 12 There is a point between the Earth and the Moon where the total gravitational field is zero. The significance of this is that returning lunar missions are able to return to Earth under the influence of the Earth's field once they pass this point. Given that the mass of Earth is $6.0 \times 10^{24} \text{ kg}$, the mass of the Moon is $7.3 \times 10^{22} \text{ kg}$ and the radius of the Moon's orbit is $3.8 \times 10^8 \text{ m}$, calculate the distance of this point from the centre of the Earth.

SOLUTIONS OF “PERID 18” QUESTIONS

A1.

$$\begin{aligned}\text{Mercury: } g &= GM/R^2 \\ &= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(3.30 \times 10^{23} \text{ kg})/(2.44 \times 10^6 \text{ m})^2 \\ &= 3.70 \text{ N kg}^{-1}\end{aligned}$$

$$\begin{aligned}\text{Saturn: } g &= GM/R^2 \\ &= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(5.69 \times 10^{26} \text{ kg})/(6.03 \times 10^7 \text{ m})^2 \\ &= 10.4 \text{ N kg}^{-1}\end{aligned}$$

$$\begin{aligned}\text{Jupiter: } g &= GM/R^2 \\ &= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1.9 \times 10^{27} \text{ kg})/(7.15 \times 10^7 \text{ m})^2 \\ &= 24.8 \text{ N kg}^{-1}\end{aligned}$$

A2.

$$\begin{aligned}\text{a } F_g &= mg = (80 \text{ kg})(3.70 \text{ N kg}^{-1}) = 296 \text{ N} \\ \text{b } F_g &= mg = (80 \text{ kg})(10.4 \text{ N kg}^{-1}) = 832 \text{ N} \\ \text{c } F_g &= mg = (80 \text{ kg})(24.8 \text{ N kg}^{-1}) = 1.98 \times 10^3 \text{ N}\end{aligned}$$

A3.

While Saturn is about 100 times more massive than Earth, this is offset by Saturn having about 10 times the radius of Earth. Gravitational field strength increases with

mass, but decreases with distance from the centre of the planet.

A4.

$$\begin{aligned}g &= (9.8 \text{ N kg}^{-1})/4^2 \\ &= 9.8 \text{ N kg}^{-1}/16 \\ &= 0.61 \text{ N kg}^{-1}\end{aligned}$$

A5.

$$\text{Acceleration} = 0.61 \text{ m s}^{-2}$$

A6.

$$\begin{aligned}\text{Acceleration at } 4.0R_E / \text{acceleration at } 2.0R_E \\ &= (2.0/4.0)^2 = 1:4 = 0.25\end{aligned}$$

A7.

$$\begin{aligned}g &= GM/R^2 \\ &= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(3.0 \times 10^{30} \text{ kg})/(10 \times 10^3 \text{ m})^2 \\ &= 2.0 \times 10^{12} \text{ N kg}^{-1}\end{aligned}$$

A8.

$$\begin{aligned}g &= GM/R^2 \\ &= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(3.0 \times 10^{30} \text{ kg})/(5.0 \times 10^6 \text{ m})^2 \\ &= 8.0 \times 10^6 \text{ N kg}^{-1}\end{aligned}$$

A9.

$$\text{Acceleration} = 8.0 \times 10^6 \text{ m s}^{-2}$$

A10.

The gravimeter would respond to differences in the gravitational field strength in a particular region due to either large deposits of low-density material (e.g. oil) or large deposits of high-density material (e.g. iron ore).

A11.

$$\begin{aligned}\text{a } g &= GM/R^2 \\ &= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(7.3 \times 10^{22} \text{ kg})/(3.0 \times 10^6 \text{ m})^2 \\ &= 0.541 \text{ N kg}^{-1} \\ F &= mg \\ &= (500 \text{ kg})(0.541 \text{ N kg}^{-1}) = 270 \text{ N} \\ \text{b } \text{Acceleration} &= 0.54 \text{ m s}^{-2} \\ \text{c } F &= mg \\ &= (50 \text{ kg})(0.541 \text{ N kg}^{-1}) = 27 \text{ N} \\ \text{d } \text{Acceleration} &= 0.54 \text{ m s}^{-2} \\ \text{e } g &= 0.54 \text{ N kg}^{-1}\end{aligned}$$

A12.

$$\begin{aligned}\text{Let } x &= \text{required distance} \\ \text{Then } 6.0 \times 10^{24} \text{ kg}/x^2 \\ &= 7.3 \times 10^{22} \text{ kg}/(3.8 \times 10^8 - x)^2 \\ \text{and } x &= 3.4 \times 10^8 \text{ m}\end{aligned}$$

HOMEWORK DUE 12 NOV THURSDAY

1 Which of the following statements is correct? A satellite in a stable circular orbit 100 km above the Earth will move:

- A with an acceleration of 9.8 m s^{-2}
- B with a constant velocity
- C with zero acceleration
- D with an acceleration of less than 9.8 m s^{-2} .

2 Explain why the gravitational field of the Earth does no work on a satellite in a stable circular orbit.

The following information applies to questions 3 and 4. The gravitational field strength at the location where the Optus D1 satellite is in stable orbit around the Earth is equal to 0.22 N kg^{-1} . The mass of this satellite is $2.3 \times 10^3 \text{ kg}$.

- 3 Using only the information given, calculate the net force acting on this satellite as it orbits.
- 4 Identify the source of this net force.
- 5 The planet Neptune has a mass of $1.02 \times 10^{26} \text{ kg}$. One of its moons, Triton, has a mass of $2.14 \times 10^{22} \text{ kg}$ and an orbital radius equal to $3.55 \times 10^8 \text{ m}$. For Triton, calculate its:
- a orbital acceleration
 - b orbital speed
 - c orbital period (in days).

9 The data for two of Saturn's moons, Atlas and Helene, is as follows. The orbit of Helene is about twice as far from the centre of Saturn as that of Atlas.

	Orbital radius (m)	Orbital period (days)
Atlas	1.37×10^8	0.602
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- a Calculate the value of these ratios:
- i orbital speed of Atlas/orbital speed of Helene
 - ii acceleration of Atlas/acceleration of Helene.
- b The largest of Saturn's moons is Titan. It has an orbital radius of $1.20 \times 10^9 \text{ m}$. Use Kepler's third law to show that the orbital period of Titan is 15.6 days.

10 The space shuttle is launched into orbit from a point A on the Equator, as shown. The shuttle then enters a stable circular orbit of radius R_o at point B. The radius of the Earth is $6.4 \times 10^6 \text{ m}$. The ratio of the gravitational field strength at A to that at B is equal to 1.2. Calculate the distance R_o .



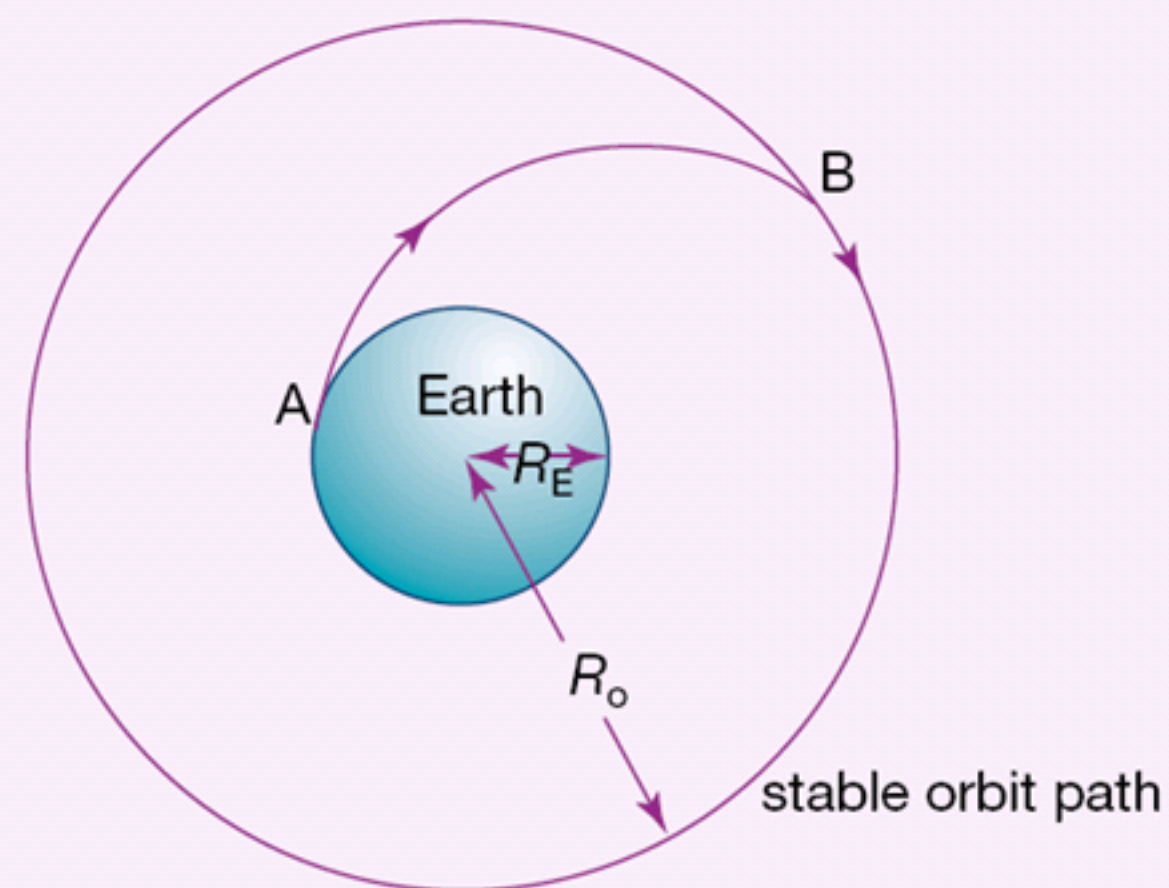
force acting on this satellite as it orbits.

- 4 Identify the source of this net force.
- 5 The planet Neptune has a mass of 1.02×10^{26} kg. One of its moons, Triton, has a mass of 2.14×10^{22} kg and an orbital radius equal to 3.55×10^8 m. For Triton, calculate its:
 - a orbital acceleration
 - b orbital speed
 - c orbital period (in days).

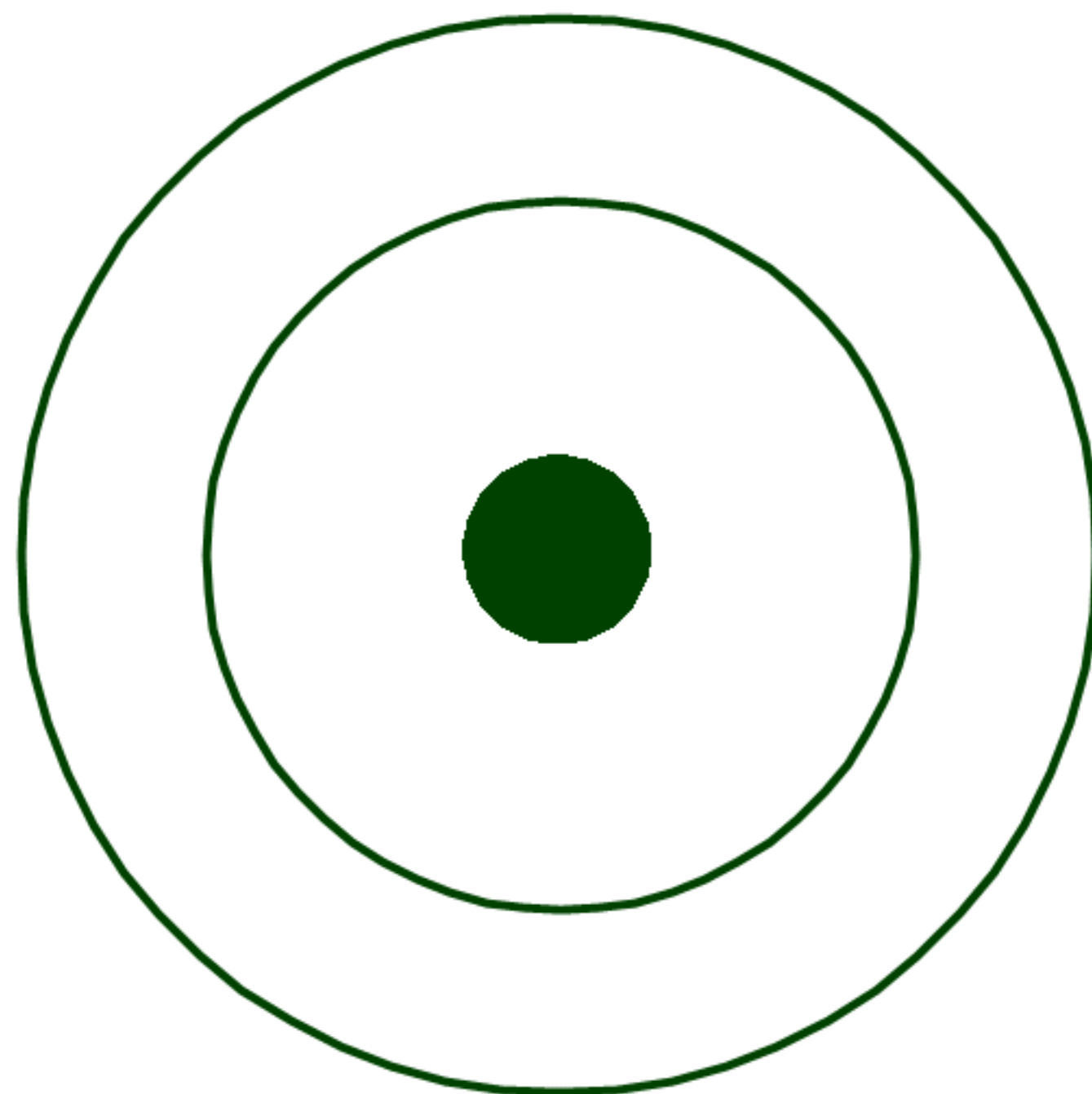
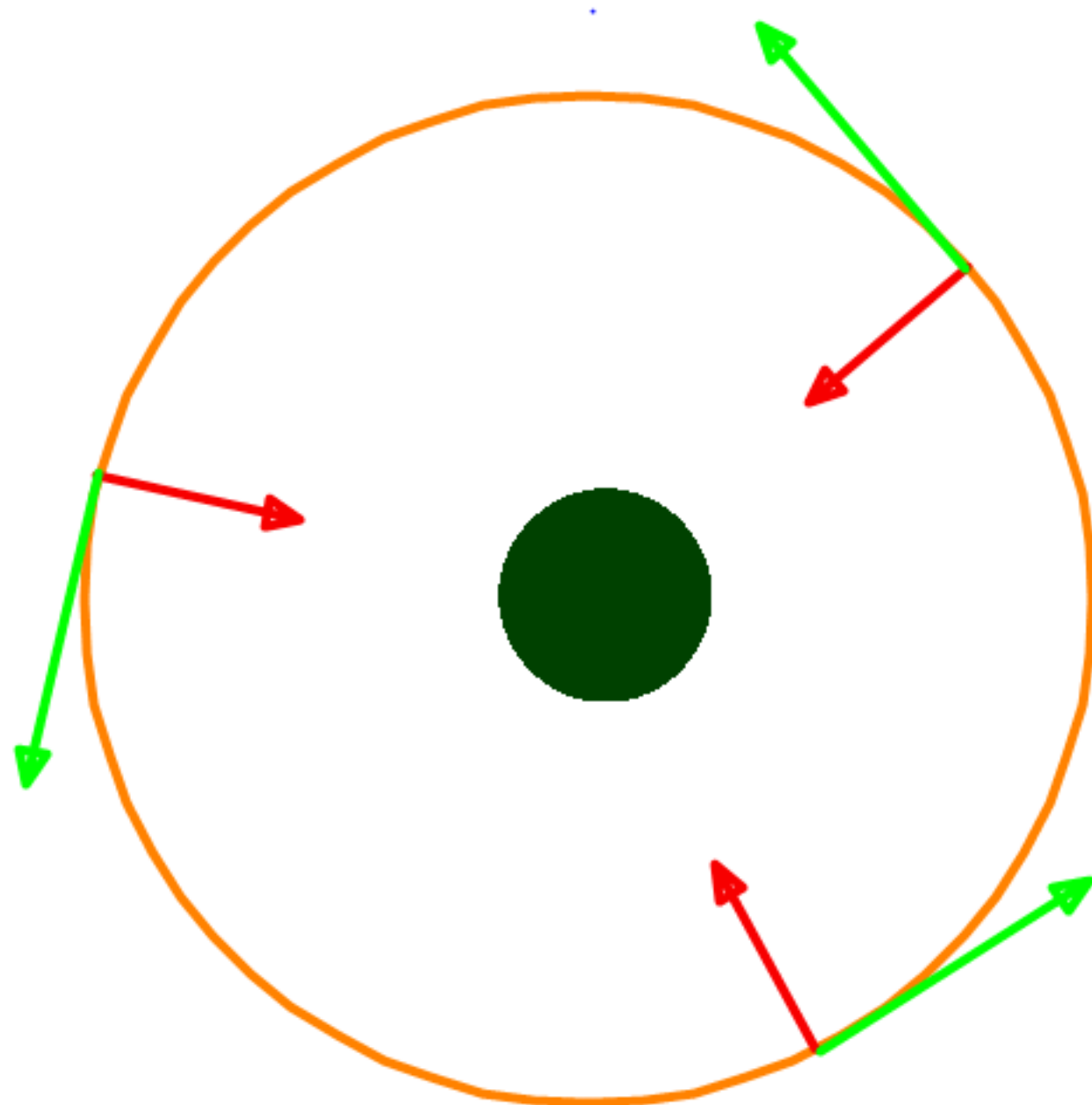
The following information applies to questions 6 and 7. One of Saturn's moons, Titan, has a mass of 1.35×10^{23} kg and an orbital radius of 1.22×10^9 m. The orbital period of Titan is 1.38×10^6 s.

- 6 Calculate the:
 - a orbital speed of Titan (in km s^{-1})
 - b orbital acceleration of Titan.
- 7 Using this data, calculate the mass of Saturn.
- 8 A satellite is in a geosynchronous orbit around the Earth if its period of rotation is the same as that of the Earth, i.e. 24 h. Such a satellite is called a geostationary satellite. Venus has a mass of 4.87×10^{24} kg and a radius of 6.05×10^6 m. The length of a day on Venus is 2.10×10^7 s. For a satellite to be in a synchronous orbit around Venus, calculate:
 - a the orbital radius of the satellite
 - b its orbital speed
 - c its orbital acceleration.

- 10 The space shuttle is launched into orbit from a point A on the Equator, as shown. The shuttle then enters a stable circular orbit of radius R_o at point B. The radius of the Earth is 6.4×10^6 m. The ratio of the gravitational field strength at A to that at B is equal to 1.2. Calculate the distance R_o .



- 11 Melbourne TV stations can use pictures from the Japanese MTSAT-1R weather satellite, which is in a geostationary orbit over the Equator to the north of Australia. Why is it not possible to place a satellite in orbit so that it is always directly over Melbourne?
- 12 Ceres, the first asteroid to be discovered, was found by Giuseppe Piazzi in 1801. Ceres has mass 7.0×10^{20} kg and radius 385 km.
 - a What is the gravitational field strength at the surface of Ceres?
 - b Determine the speed required by a satellite in order to remain in orbit 10 km above the surface of Ceres.



Space 2

The Solar System is held together by gravity.

Students learn to:

- describe a gravitational field in the region surrounding a massive object in terms of its effects on other masses in it
- define Newton's Law of Universal Gravitation

$$F = G \frac{m_1 m_2}{d^2}$$

- discuss the importance of Newton's Law of Universal Gravitation in understanding and calculating the motion of satellites
- identify that a slingshot effect can be provided by planets for space probes

Students:

- present information and use available evidence to discuss the factors affecting the strength of the gravitational force
- solve problems and analyse information using

$$F = G \frac{m_1 m_2}{d}$$

Steps in solving PM questions.

Step 1 > Read the question.

Step 2 > Understand the question.

Step 3 > Make sure you understand "What is given/provided" and "What is asked".

Step 4 > Draw a diagram.

Step 5 > Select your interval (A to B). Mark A and B on your diagram.

Step 6 > Draw the data table and fill in the details as much as you can. Mark unknowns.

Step 7 > Select the appropriate formula and solve it for unknowns.

