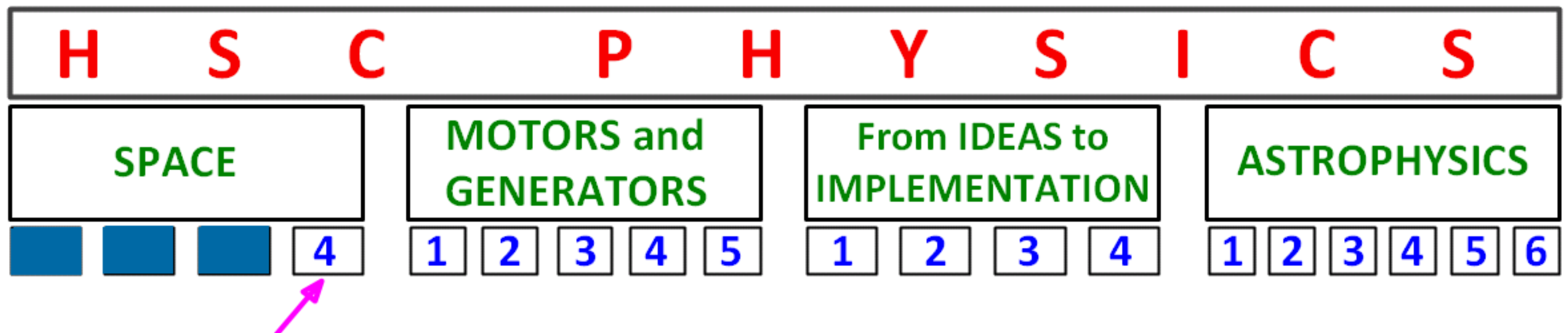


SPACE

1st Quarter; Module 1

PERIOD 23 - the day you have been waiting for!

Constancy of Speed of Light, Time Dilation



Space 4

Current and emerging understanding about time and space has been dependent upon earlier models of the transmission of light

Students learn to:

- outline the features of the aether model for the transmission of light
- describe and evaluate the Michelson-Morley attempt to measure the relative velocity of the Earth through the aether
- discuss the role of the Michelson-Morley experiments in making determinations about competing theories
- outline the nature of inertial frames of reference
- discuss the principle of relativity
- describe the significance of Einstein's assumption of the constancy of the speed of light
- identify that if c is constant then space and time become relative
- discuss the concept that length standards are defined in terms of time in contrast to the original metre standard
- explain qualitatively and quantitatively the consequence of special relativity in relation to:
 - the relativity of simultaneity
 - the equivalence between mass and energy
 - length contraction
 - time dilation
 - mass dilation
- discuss the implications of mass increase, time dilation and length contraction for space travel

Space 4

Current and emerging understanding about time and space has been dependent upon earlier models of the transmission of light

Students:

- gather and process information to interpret the results of the Michelson-Morley experiment
- perform an investigation to help distinguish between non-inertial and inertial frames of reference
- analyse and interpret some of Einstein's thought experiments involving mirrors and trains and discuss the relationship between thought and reality
- analyse information to discuss the relationship between theory and the evidence supporting it, using Einstein's predictions based on relativity that were made many years before evidence was available to support it
- solve problems and analyse information using:

$$E = mc^2$$

$$l_v = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

RELATIVE MOTION

Is it possible for one car travelling at 100 km/h to collide with another car travelling at 99 km/h and there to be no serious damage or injuries? You might have realised that this could well be the case as long as the two cars are travelling in the same direction. The amount of damage that results from a collision depends not so much on how fast the cars are moving, but on what their velocity is relative to each other. If the two cars had collided while travelling in opposite directions, the consequences would have been catastrophic for the cars and their occupants.

When you discuss the velocity of an object, you usually assume, without saying so, that the frame of reference is the Earth. If you describe an emu as walking with a constant velocity, you really mean that the emu has a constant velocity relative to Earth. In the past, this omission has not been an issue because the frame of reference has usually been Earth. In situations where we are analysing motion from a different frame of reference, the frame needs to be stated.

Imagine that you are in train A that is moving east with a constant speed of 10 m/s and you are walking along the aisle towards the front of the train at 2 m/s. What velocity are you travelling at? The answer to this question is that it depends on which frame of reference you are using. To a person sitting in the train, your velocity is 2 m/s east. Here, your frame of reference is the moving train. However, to a person standing on the station platform, your velocity will be 12 m/s east. In this case, your frame of reference is the ground or Earth. Now imagine that you sit down and that another train, train B, passes in the opposite direction at 5 m/s relative to the ground. What is the velocity of this train relative to your train?

The velocity of train B relative to train A gives the apparent motion of train B when seen from train A. In other words, this gives train A's view of how fast train B seems to be moving. From your frame of reference in train A, train B seems to be travelling to the west but faster than 5 m/s. To calculate the relative velocity of two objects, we need to perform a vector subtraction.

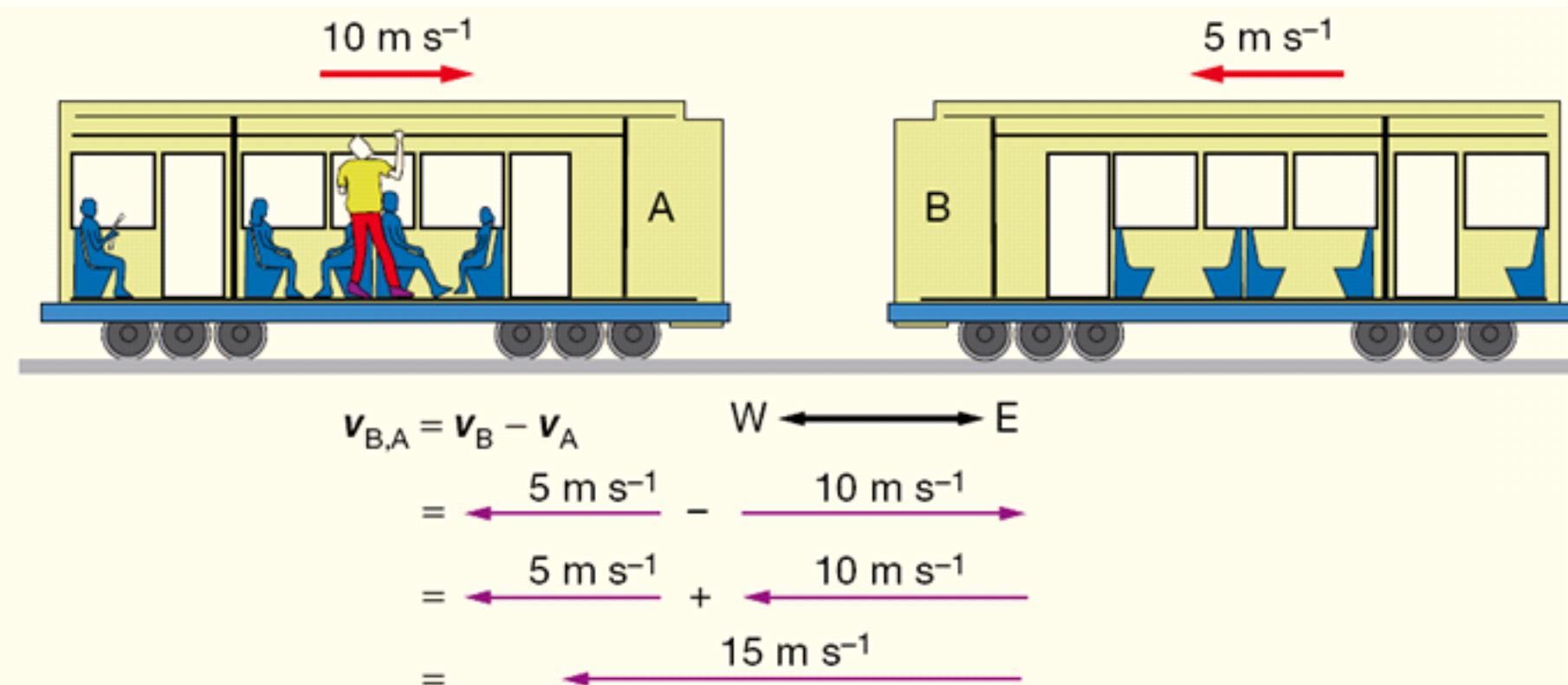


Figure 1.8 To determine the velocity of train B relative to train A, a vector subtraction is performed, giving a relative velocity of 15 m s^{-1} west.

FRAMES OF REFERENCE

Inertial Frames of Reference
(**IFR**)

IFRs are either moving
at constant velocity

or Stationary

$$v = \text{constant}$$

OR

$$v = 0$$

non-Inertial
Frames of Reference

nIFRs ~~are~~ have
acceleration,
they are accelerating
frames of reference.

The Principle of Relativity

first stated by Galileo, around 1600s and embodied in Newton's first law

Briefly it states that "All steady motion is relative and cannot be detected without reference to an outside point"

Put another way, if you are travelling inside a vehicle you cannot tell if you are moving at a steady velocity or standing still without looking out the window.

You may have experienced this personally when sitting in a train and an adjacent train begins to roll — at first you may think that your own train is moving until you look out of a window on the other side of the carriage.

The Principle of Relativity states that

"Within an inertial frame of reference (IFR) you cannot perform any mechanical experiment or observation that would reveal to you whether you were moving with uniform velocity or standing still"

or

"It is not possible to perform an experiment within an inertial frame of reference to detect the motion of the frame of reference"

The Principle of Relativity

first stated by Galileo, around 1600s and embodied in Newton's first law

Put another way, if you are travelling inside a vehicle you cannot tell if you are moving at a steady velocity or standing still without looking out the window.

You may have experienced this personally when sitting in a train and an adjacent train begins to roll — at first you may think that your own train is moving until you look out of a window on the other side of the carriage.

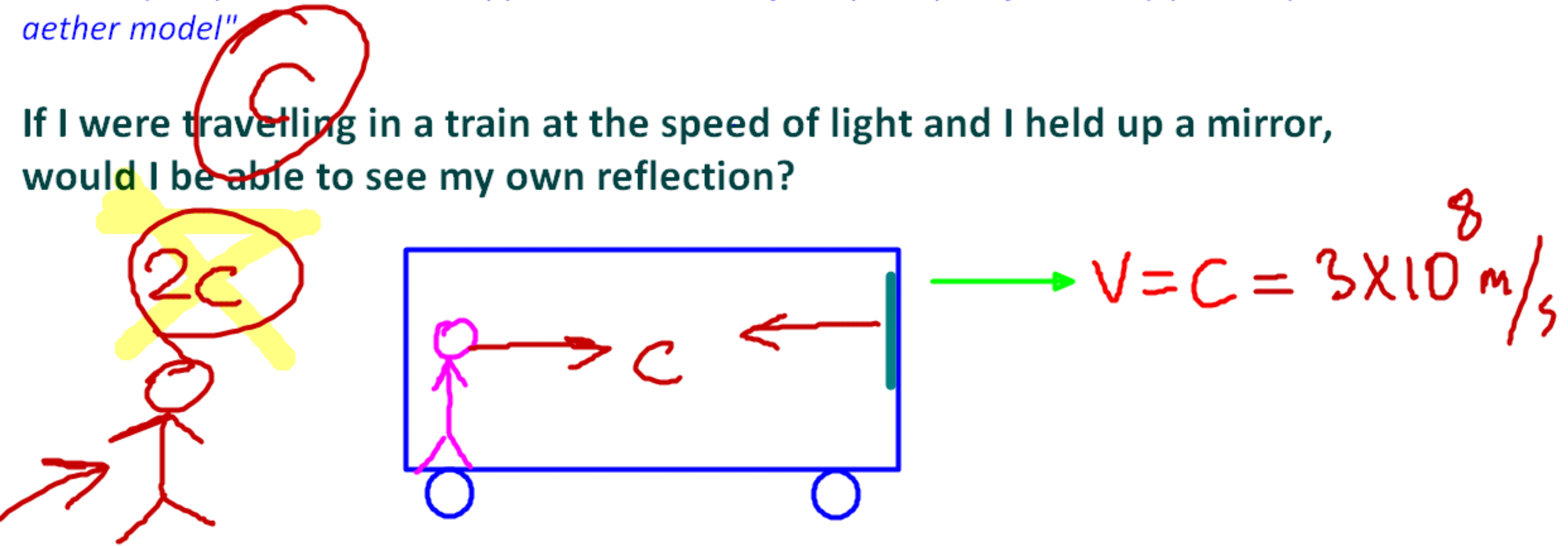
The Principle of Relativity states that

or

EINSTEIN'S QUESTION

"The way he puzzled over the apparent violation of the principle of relativity posed by the aether model"

If I were travelling in a train at the speed of light and I held up a mirror, would I be able to see my own reflection?



NO > If the aether model was right, light could go no faster than the train. It could never catch up with the mirror to return as a reflection. The principle of relativity is thus violated because seeing one's reflection disappear would be a way to detect motion.

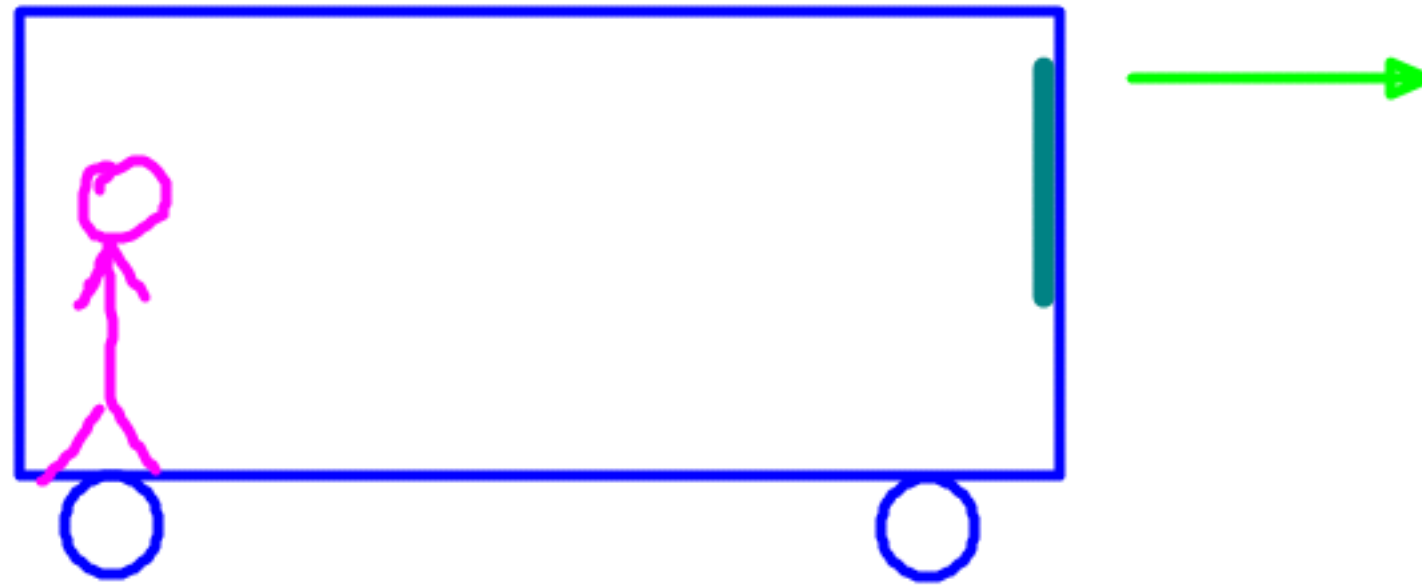
YES > On the other hand, if the principle of relativity were not to be violated, the reflection must be seen normally, which means that it is moving away from the mirror holder at $3 \times 10^8 \text{ m/s}$. However, this would mean that an observer on the embankment next to the train would see that light travelling at twice its normal speed.



EINSTEIN'S QUESTION

"The way he puzzled over the apparent violation of the principle of relativity posed by the aether model"

If I were travelling in a train at the speed of light and I held up a mirror, would I be able to see my own reflection?



NO >

YES >

Einstein said:

Yes, I would see my reflection; The aether model must be wrong and the speed of light is constant regardless of the motion of the observer

His Theory of Special Relativity:

- ★ The laws of physics are the same in all frames of reference; that is, **the principle of relativity always holds**
- ★ **The speed of light is independent of the motion of the observer**; that is, everyone always observes the same speed of light regardless of their motion

Also: The luminiferous aether is superfluous (unnecessary); that is, it is no longer needed to explain the behaviour of light.

Einstein said:

Yes, I would see my reflection; The aether model must be wrong and the speed of light is constant regardless of the motion of the observer

His Theory of Special Relativity:



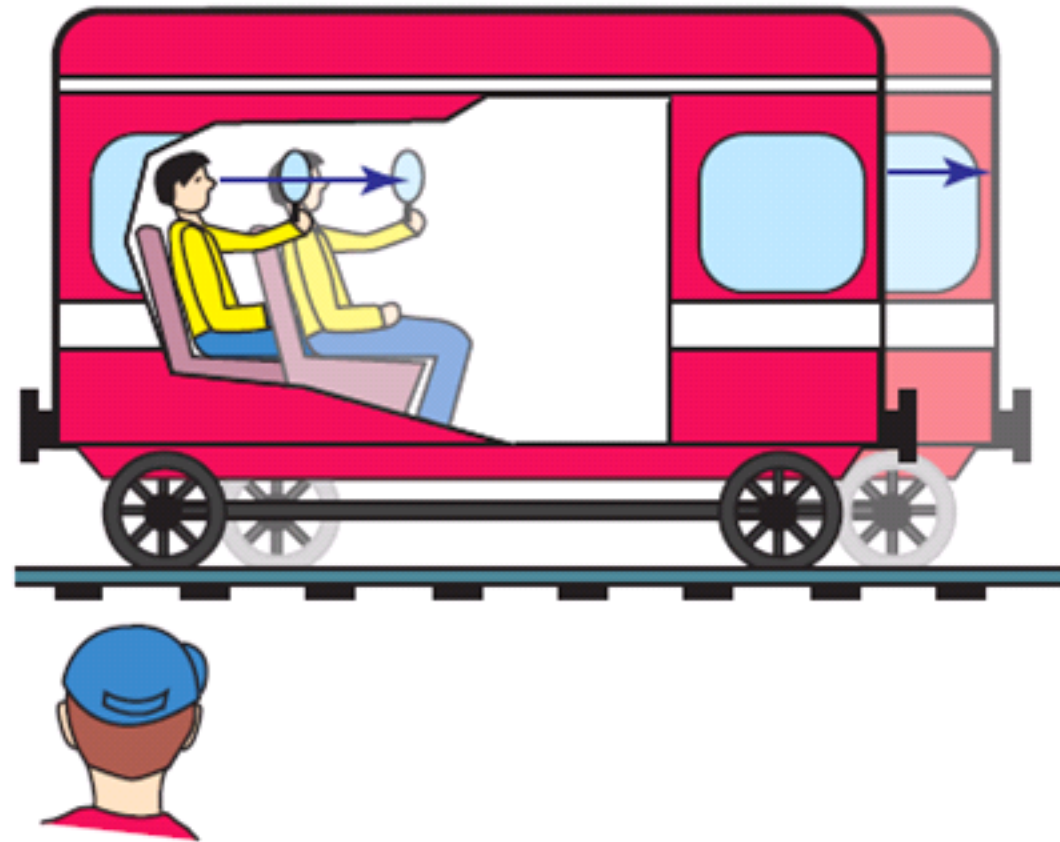
Also:

The significance of Einstein's assumption of constancy of the speed of light

In Newtonian physics, distance and velocity can be relative terms, but **time is an absolute and fundamental quantity**.

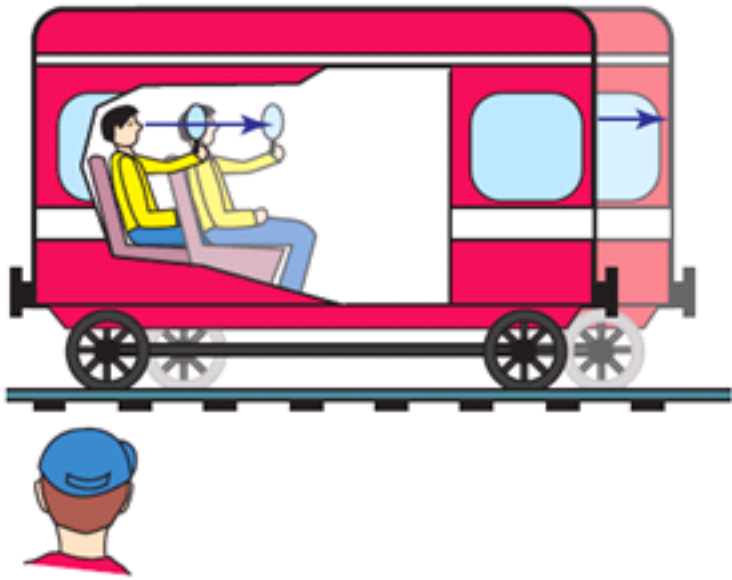
Figure uses the example of the train rider with the mirror, and shows how the velocity of the light is characterised by two events (if we consider just the forward part of the light's journey).

- ✦ the light leaving his face and
- ✦ the light arriving at the mirror



Remembering that the train is travelling at the speed of light, Newtonian physics says that the observer on the embankment outside the train records **precisely double the distance** of the journey of the light compared with that recorded by the train rider; however, **they both record the same time**. Since velocity, $v = \frac{d}{t}$, this means that the observer on the embankment would measure a velocity of light twice that measured by the train rider.

The significance of Einstein's assumption of constancy of the speed of light



- ✦ According to Einstein's theory both the observer on the embankment and the train rider will measure the **same value for the velocity of light 'c'**.
- ✦ This could only be true if **the observer and the rider observed different times as well as different distances** in such a way that distance divided by time always equals the same value, c .
- ✦ Einstein radically altered the assumptions of Newtonian physics so that now **the speed of light is absolute, and space and time are both relative quantities** that depend upon the motion of the observer.
- ✦ In other words, the measured length of an object and the time taken by an event **depend entirely upon the velocity of the observer**.
- ✦ Further to this, since neither space nor time are absolute, the theory of relativity has replaced them with the concept of a space–time continuum. Any event then has four dimensions (three space coordinates plus a time coordinate) that fully define its position within its frame of reference.

NEWTON vs EINSTEIN

NEWTON >

Imagine that you are in a car that is travelling along a straight, flat stretch of freeway at 100 km/h. You have an apple in your hand that you toss up and down and side to side. In the car, we can say that your frame of reference is moving with a constant velocity relative to the ground. Now, say that you repeat exactly the same actions on the apple when the car is stationary. The motion of the apple, its mass and acceleration are exactly the same in this stationary reference frame as they were when the car was moving with constant velocity.

A stationary frame of reference and a frame of reference with constant velocity are called inertial frames of reference. Newton's laws of motion are valid in these inertial frames of reference. Newton also assumed that physical quantities such as mass, time, distance and so on were absolute quantities. This means that their values did not change whatever the frame of reference. This would seem to make sense. After all, the mass of an apple and the length of a metre ruler don't change as they travel faster-or do they?

EINSTEIN >

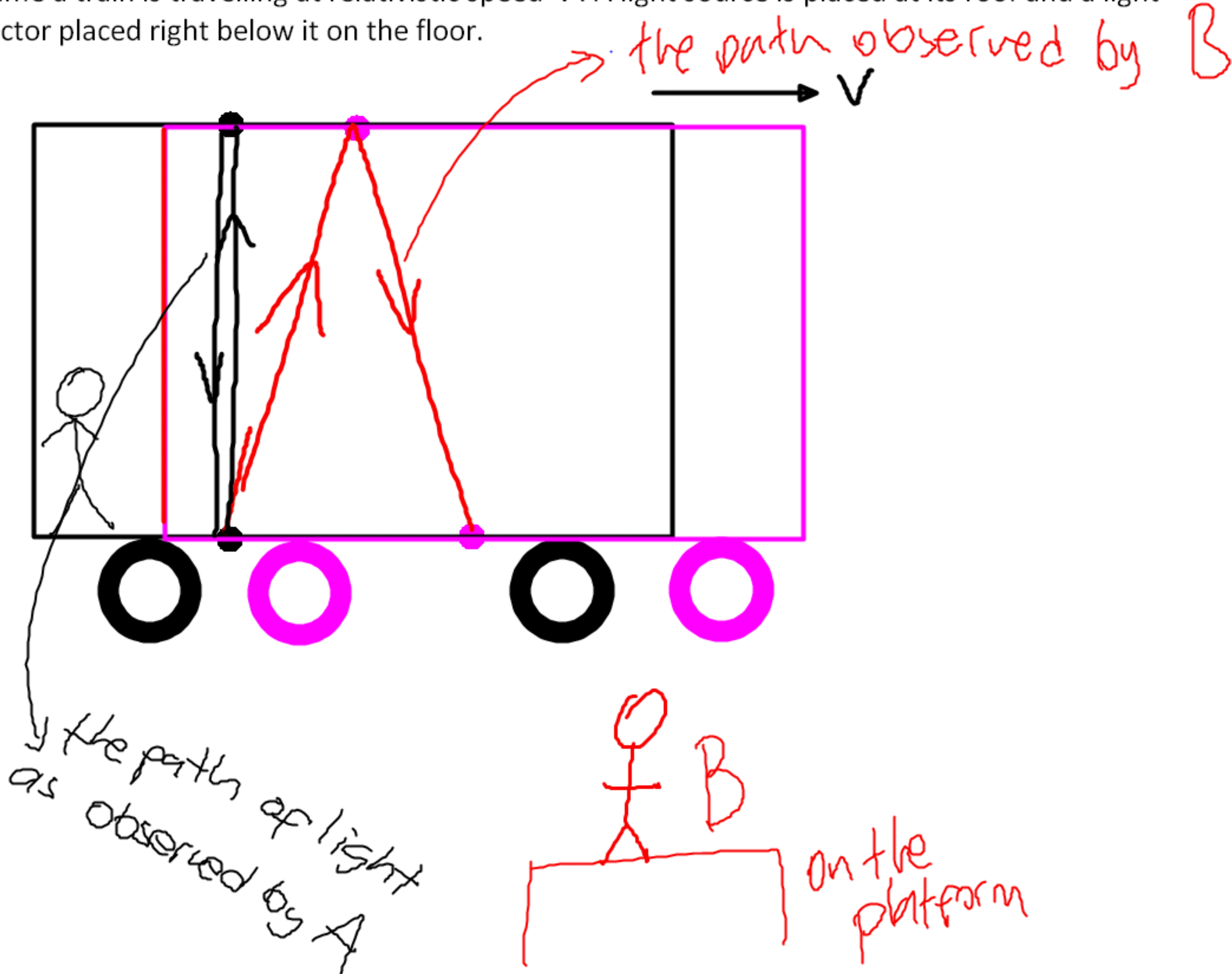
About 200 years after Newton published his laws of motion, Albert Einstein showed that Newton's laws did not work at speeds approaching the speed of light. In fact, at these high speeds, the mass of an object is greater, time slows down and lengths shrink! These ideas are outlined in Einstein's theory of special relativity. In this theory, Newton's ideas of the absolute nature of space and time were replaced by Einstein's ideas of the relative nature of space and time. In fact, history has shown that Newton's laws were a special case of Einstein's theories, applying only to situations involving comparatively slow-moving objects.

Time Dilation

The time measured for the same incident depends on the speed of the observer! - How come?

Another "thought experiment"

Assume a train is travelling at relativistic speed ' v '. A light source is placed at its roof and a light detector placed right below it on the floor.



$$(vt_B)^2 + (ct_A)^2 = (c \cdot t_B)^2$$

$$t_B = \frac{t_A}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$



$$t_v = \frac{t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

if $v \ll c$ then $\frac{v}{c} \rightarrow 0 \Rightarrow t_v = t_0$

if $v \rightarrow c$ then $\frac{v}{c} \rightarrow 1 \Rightarrow t_v \gg t_0$

Example 1: A time-dilated sneeze

A train traveller sneezes just as his train passes through a station. The sneeze takes precisely 1.000 s as measured by another person seated next to the sneezer. If the train is travelling at half the speed of light, how long does the sneeze take as seen by a person standing on the platform of the station?

Example 2: A time-dilated yawn

Continuing the last problem, if the person standing on the platform yawned just as the train was passing through, and this yawn lasted 2.000 s as measured by the yawner, what would be the duration of the yawn as measured by the train travellers?

Proofs of Time Dilation

1. Comparison of atomic clocks that have been flown over long journeys with clocks that have remained stationary for the same period. The travelling clock was found to be lagging slightly behind the the one stayed stationery when they are brought together again. These experiments are possible only because of the extreme accuracy of atomic clocks built over the last few decades, even though Einstein predicted this effect about 100 years ago.

2. Further supporting evidence has been found in the **abundance of mesons striking the ground** after having been created in the upper atmosphere by incoming cosmic rays. What is surprising is that the mesons have a velocity of about $0.996c$ and, at that speed, should take approximately 16 microseconds to travel through the atmosphere. However, when measured in a laboratory, mesons have an average lifetime of approximately 2.2 microseconds. This anomaly can be explained by the fact that 2.2 microseconds represents their proper lifetime, as measured in their rest frame, whereas 16 microseconds is a dilated lifetime due to their relativistic speed.

PERIOD 22 SOLUTIONS

A1.

- a $x = ut + 0.5at^2$
then $4.9 \text{ m} = 0 + 0.5(9.8 \text{ m s}^{-2})t^2$
and $t = 1.0 \text{ s}$
- b $x = (\text{average speed})(\text{time}) = (20 \text{ m s}^{-1})(1.0 \text{ s}) = 20 \text{ m}$
- c The acceleration of the ball is constant at any time during its flight, and is equal to the acceleration due to gravity $= 9.8 \text{ m s}^{-2}$ down
- d After 0.80 s , the ball has two components of velocity:
 $v_x = 20 \text{ m s}^{-1}$
and $v_y = 0 + (9.8 \text{ m s}^{-2})(0.80 \text{ s}) = 7.84 \text{ m s}^{-1}$
The speed of the ball at 0.80 s is given by:
 $[(20 \text{ m s}^{-1})^2 + (7.84 \text{ m s}^{-1})^2]^{\frac{1}{2}} = 21.5 \text{ m s}^{-1}$
- e The ball will strike the ground 1.0 s after it is struck.
Then $v_x = 20 \text{ m s}^{-1}$
and $v_y = 0 + (9.8 \text{ m s}^{-2})(1.0 \text{ s}) = 9.8 \text{ m s}^{-1}$
The speed of the ball at 1.0 s is given by:
 $[(20 \text{ m s}^{-1})^2 + (9.8 \text{ m s}^{-1})^2]^{\frac{1}{2}} = 22.3 \text{ m s}^{-1}$

A2.

- a The horizontal velocity of the ball remains constant and $v_x = 10 \text{ m s}^{-1}$.
- b $v^2 = u^2 + 2ax$
and $v_y^2 = 0^2 + 2(9.8 \text{ m s}^{-2})(1.0 \text{ m})$
and $v_y = 4.4 \text{ m s}^{-1}$ down
- c $v = [(10 \text{ m s}^{-1})^2 + (4.43 \text{ m s}^{-1})^2]^{\frac{1}{2}} = 10.9 \text{ m s}^{-1}$ at 24° to the horizontal,
where the angle is determined from

$$\tan \theta = 4.43 \text{ m s}^{-1} / 10 \text{ m s}^{-1} = 0.443 \text{ and } \theta = 24^\circ$$

- d $x = ut + 0.5at^2$
and $1.0 \text{ m} = 0 + 0.5(9.8 \text{ m s}^{-2})t^2$
so $t = 0.45 \text{ s}$
- e Horizontal distance = (horizontal speed)(time) $= (10 \text{ m s}^{-1})(0.45 \text{ s}) = 4.5 \text{ m}$

A3.

- a $v_x = (28 \text{ m s}^{-1}) \cos 30^\circ = 24.2 \text{ m s}^{-1}$ north
and remains constant throughout the flight.
- b 24.2 m s^{-1} north
- c 24.2 m s^{-1} north

A4.

- a $v_y = (28 \text{ m s}^{-1}) \sin 30^\circ = 14 \text{ m s}^{-1}$ up
- b $v_y = 14 \text{ m s}^{-1} - (9.8 \text{ m s}^{-2})(1.0 \text{ s}) = 4.2 \text{ m s}^{-1}$ up
- c The time for the ball to reach its maximum height is determined from $v = u + at$.
Then at maximum height, the vertical velocity of the ball $= 0$
and $0 = 14 \text{ m s}^{-1} - (9.8 \text{ m s}^{-2})t$
and $t = 1.43 \text{ s}$
Therefore at $t = 2.0 \text{ s}$ the ball is 0.57 s into its downward flight.
 $v_y = 0 + (9.8 \text{ m s}^{-2})(0.57 \text{ s}) = 5.6 \text{ m s}^{-1}$ down

A5.

- a The time for the ball to reach its maximum height is determined from $v = u + at$.
Then at maximum height, the vertical velocity of the ball $= 0$
and $0 = 14 \text{ m s}^{-1} - (9.8 \text{ m s}^{-2})t$
and $t = 1.43 \text{ s}$
- b $v^2 = u^2 + 2ax$
then $0 = (14 \text{ m s}^{-1})^2 - (9.8 \text{ m s}^{-2})x$
and $x = 10 \text{ m}$
- c The acceleration of the ball is constant at any time during its flight, and is equal to the acceleration due to gravity $= 9.8 \text{ m s}^{-2}$ down.

A6.

- a The minimum speed will occur when the vertical components of the ball's velocity $= 0$, i.e. at the maximum height.
- b The minimum velocity of the ball during its flight occurs at the maximum height, and is equal to the horizontal component of the ball's velocity $= 24.2 \text{ m s}^{-1}$ horizontally.
- c The minimum speed of the ball during its flight occurs at the maximum height at $t = 1.43 \text{ s}$.

A7.

- a The flight of the ball is symmetrical. Therefore the time for it to reach the ground after launching $= 2(1.43 \text{ s}) = 2.86 \text{ s}$.

b The flight of the ball is symmetrical.
Therefore the ball will strike the ground at the same velocity as that when it was launched: 28 m s^{-1} at an angle of 30° to the horizontal.

c Horizontal range = (horizontal speed)(time)
 $= (24.2 \text{ m s}^{-1})(2.86 \text{ s}) = 69.2 \text{ m}$

A8.

C is the correct answer. Air resistance is a force that would be acting in the opposite direction to the horizontal velocity of the ball, thereby producing a horizontal deceleration of the ball during its flight.

A9.

a The initial E_K of the ball $= 0.5(0.250 \text{ kg})(16 \text{ m s}^{-1})^2 = 32 \text{ J}$

At its maximum height $E_K = 16 \text{ J}$,
 then loss in $E_K = 32 \text{ J} - 16 \text{ J} = 16 \text{ J} = (0.250 \text{ kg})(9.8 \text{ m s}^{-2})h$
 and maximum height $h = 6.53 \text{ m}$

b $v^2 = u^2 + 2ax$
 and $0 = u^2 - 2(9.8 \text{ m s}^{-2})(6.5306 \text{ m})$
 then the initial vertical velocity $u = 11.3 \text{ m s}^{-1}$ up

c At its maximum height the velocity of the ball is equal to its horizontal component v_x and $E_K = 16 \text{ J} = 0.5(0.250 \text{ kg})v_x^2$
 and $v_x = 11.31 \text{ m s}^{-1} = (16 \text{ m s}^{-1}) \cos \theta$
 then $\cos \theta = 0.70625$ and $\theta = 45^\circ$

d $v_x = (16 \text{ m s}^{-1}) \cos 45^\circ = 11.31 \text{ m s}^{-1}$
 $v_y = 11.31 - (9.8 \text{ m s}^{-2})(1.0 \text{ s}) = 1.51 \text{ m s}^{-1}$

then the speed of the ball at $t = 1.0 \text{ s}$
 $= [(11.31 \text{ m s}^{-1})^2 + (1.51 \text{ m s}^{-1})^2]^{1/2} = 11.4 \text{ m s}^{-1}$

e The horizontal displacement of the ball at $t = 1.0 \text{ s}$
 $= (11.31 \text{ m s}^{-1})(1.0 \text{ s}) = 11.31 \text{ m}$,
 while the vertical displacement at $t = 1.0 \text{ s}$
 $= (11.31 \text{ m s}^{-1})(1.0 \text{ s}) - 0.5(9.8 \text{ m s}^{-2})(1.0 \text{ s})^2 = 6.41 \text{ m}$

The resultant displacement after $1.0 \text{ s} = [(11.31 \text{ m})^2 + (6.41 \text{ m})^2]^{1/2} = 13.0 \text{ m}$

The angle θ of the displacement from the horizontal is given by:

$\tan \theta = 6.41 \text{ m} / 11.31 \text{ m} = 0.5668$
 and $\theta = 30^\circ$

f $v = u + at$
 then at maximum vertical height $v_y = 0$
 therefore $0 = 11.31 \text{ m s}^{-1} - (9.8 \text{ m s}^{-2})t$
 and $t = 1.154 \text{ s}$
 Since flight is symmetrical, time of flight $T = 2(1.154 \text{ s}) = 2.31 \text{ s}$.

g Horizontal distance = (horizontal speed)(time) $= (11.31 \text{ m s}^{-1})(2.31 \text{ s}) = 2.61 \text{ m}$

A10.

a Taking down as positive and the top of the ramp as the zero position. Using the vertical component and finding initial velocity by analysing motion to maximum height:

$a = 9.80 \text{ m s}^{-2}$, $t = 1.5 \text{ s}$, $v = 0$, $u = ?$

$v = u + at$

$0 = u + 9.80 \times 1.50$

$u_y = -14.7 \text{ m s}^{-1}$; i.e. 1.47 m s^{-1} up

Trigonometry can be used to determine initial speed: $v = u_y / \sin 40^\circ = 14.7 / 0.643 = 22.9 \text{ m s}^{-1}$

b Using vertical component: $a = 9.80 \text{ m s}^{-2}$, $t = 1.50 \text{ s}$, $u = -14.7 \text{ m s}^{-1}$, $v = 0$, $x = ?$

$x = \frac{1}{2}(u + v)t = 0.5 \times -14.7 \times 1.50 = 11.0 \text{ m}$

c Using vertical component: $u = -14.7 \text{ m s}^{-1}$, $a = 9.8 \text{ m s}^{-2}$, $x = 10 \text{ m}$, $t = ?$

First find final vertical velocity (to avoid quadratic equation): $v = ?$

$v^2 = u^2 + 2ax = (-14.7)^2 + 2 \times 9.80 \times 10 = 412$

$v = 20.3 \text{ m s}^{-1}$

Now find total flight time: $t = ?$

$v = u + at$

$20.3 = -14.7 + 9.80 \times t$

$t = 3.57 \text{ s}$



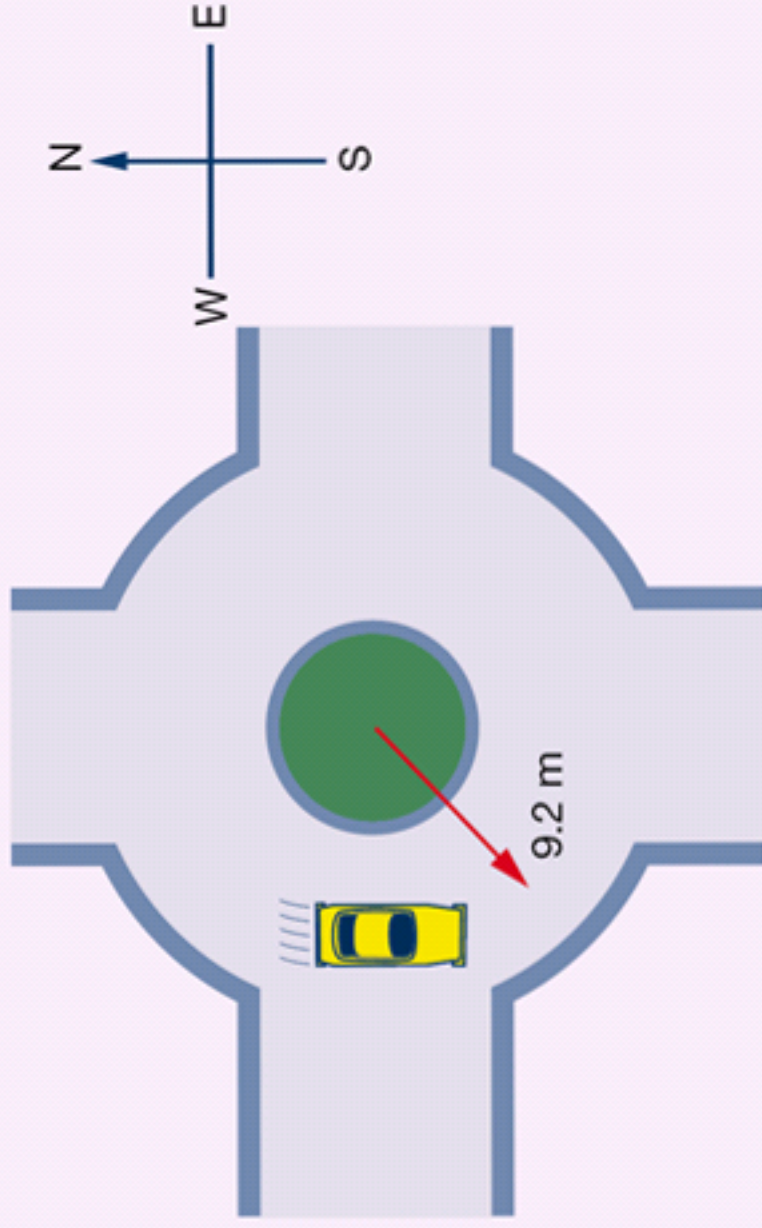
2.5 questions

Circular motion

In the following questions, assume that the acceleration due to gravity is 9.80 m s^{-2} and ignore the effects of air resistance.

The following information applies to questions 1–5.

A car of mass 1200 kg is travelling on a roundabout in a circular path of radius 9.2 m . The car moves with a constant speed of 8.0 m s^{-1} .



1 a Which two of the following statements correctly describe the motion of the car as it travels around the roundabout?

- A** It has a constant speed.
 - B** It has a constant velocity.
 - C** It has zero acceleration.
 - D** It has an acceleration that is directed towards the centre of the roundabout.
- b** As the car turns towards the left, a passenger describes the effect on her as 'being thrown across towards the right side of the cabin'. What has actually happened?

2 When the car is in the position shown in the diagram:

- a** what is the speed of the car?
- b** what is the velocity of the car?
- c** what is the magnitude and direction of the acceleration of the car?

3 a Calculate the magnitude and direction of the net force acting on the car at the position shown.

b Identify the force that is enabling the car to move in its circular path.

4 Some time later, the car has travelled halfway around the roundabout. What is:

- a** the velocity of the car at this point?
- b** the direction of its acceleration at this point?

5 If the driver of the car kept speeding up, what would eventually happen to the car as it travelled around the roundabout? Explain.

6 An ice skater of mass 50 kg is skating in a horizontal circle of radius 1.5 m at a constant speed of 2.0 m s^{-1} .

- a** What is the acceleration of the skater?
- b** Are the forces acting on the skater balanced or unbalanced? Explain.
- c** Calculate the magnitude of the net force acting on the skater.
- d** Identify the force that is enabling the skater to move in a circular path.

7 An athlete competing at a junior sports meet swings a 2.5 kg hammer in a horizontal circle of radius 0.80 m at 2.0 revolutions per second. Assume that the wire is horizontal at all times.

- a** What is the period of rotation of the ball?
- b** What is the orbital speed of the ball?
- c** Calculate the acceleration of the ball.
- d** What is the magnitude of the net force acting on the ball?
- e** Name the force that is responsible for the centripetal acceleration of the ball.
- f** Describe the motion of the ball if the wire breaks.

The following information applies to questions 8–10. Frank and Col are flying their remote-controlled model plane. It has a mass of 1.6 kg and travels in a horizontal circular path of radius 62 m with a speed of 50 km h^{-1} . The plane is controlled by a radio transmitter so there are no strings attached.

- 8** Calculate the period of its motion.
- 9** Determine the magnitude of the net force that is acting on the plane.
- 10** Discuss the nature of the force that is enabling the plane to move in a circular path.



HOMEWORK

- ✦ Homework is an integral part of your "Learning Curve", take it seriously!
- ✦ Target minimum 1 hour of Physics everyday
- ✦ Divide your physics home study in three segments;
 - ✓ Revision (past)
 - ✓ Homework (present)
 - ✓ Tomorrow (future)
- ✦ Homework is due next period, unless otherwise stated
- ✦ If you cannot do all, at least do a few from each piece

*Apart from **reading the relevant pages from the textbook and solving the rest of the questions in this booklet** your homework is:*

- ✓ Study CSU Space 4 notes
- ✓ 10 questions in this booklet
- ✓ Relevant pages in Multiple Choice Dot Points Book (DPB)
- ✓ New Dot Points booklet (pages 24-27)
- ✓ Chapter 4 all questions
- ✓ 10 questions of P22
- ✓ 20 questions of P21
- ✓ 12 questions of P19
- ✓ 12 questions of P18
- ✓ 8 questions of P17
- ✓ Experiment 5 Report
- ✓ Chapter 3 all questions

NEXT PERIOD > OTHER RESULTS OF CONSTANCY OF SPEED OF LIGHT