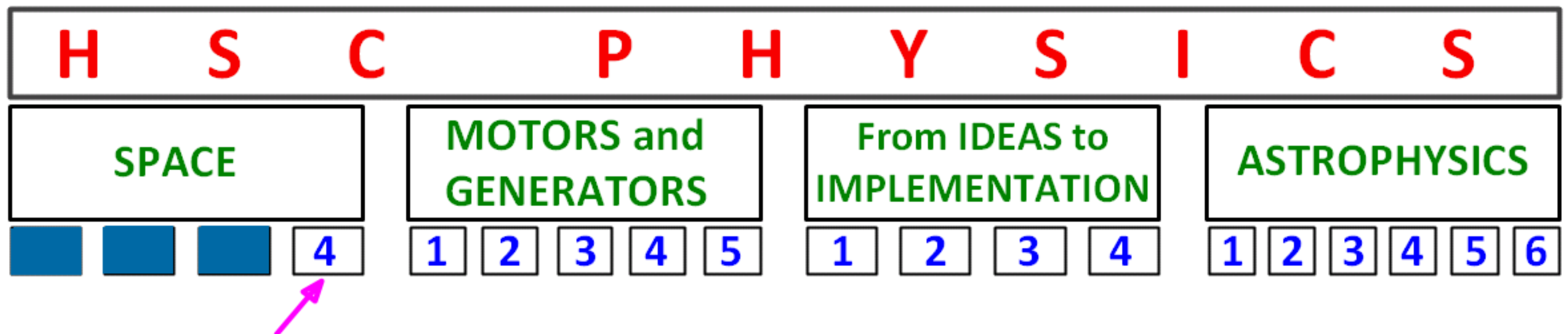


SPACE

1st Quarter; Module 1

PERIOD 25

Time Dilation, Length Contraction, Relativity of Simultaneity



STEPS FOR RELATIVITY PROBLEMS

1. Read and Understand the question
2. Identify the two IFRs
3. Decide what is being measured [What is the incident?]
4. Decide which IFR the incident belongs to. Label the two IFRs as "proper (rest) [t_o]" and "travelling [t_v]"
5. Check if the relative velocity of IFRs is provided.
6. Pick the appropriate formula and solve for unknown.

TIME DILATION

> a result of 'constancy of speed of light'

$$t_v > t_0$$

t_0 = time taken in the rest frame of reference
= proper time or rest time

t_v = time taken as seen from the frame of reference
in relative motion to the rest frame

v = velocity of the train

c = speed of light.

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time dilation is the slowing down of events as observed from a reference frame in relative motion.

Time dilation can be generally stated as follows: The time taken for an event to occur within its own rest frame is called the proper time t_0 , or rest time. Measurements of this time, t_v , made from any other inertial reference frame in relative motion to the first, are always greater.

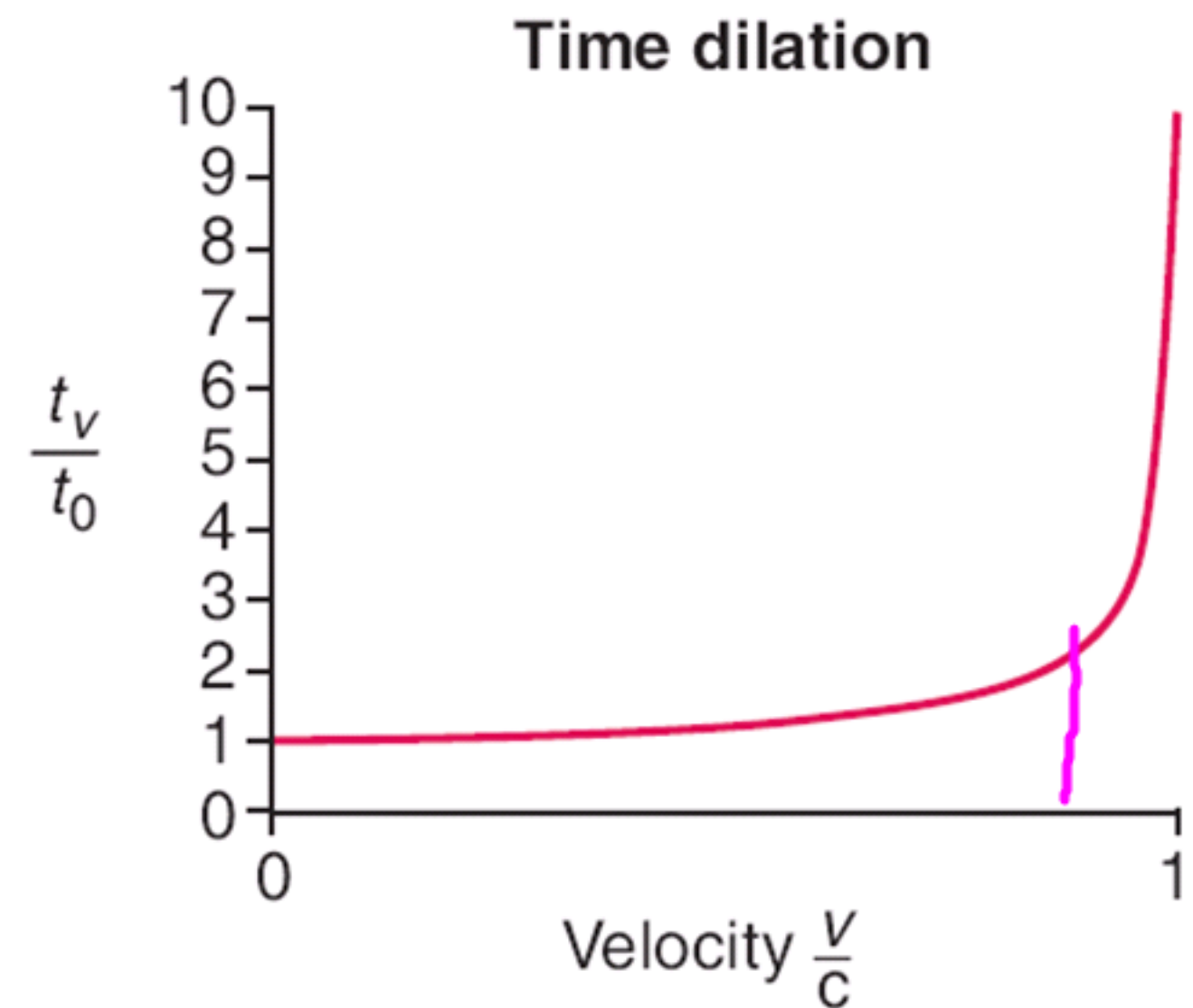
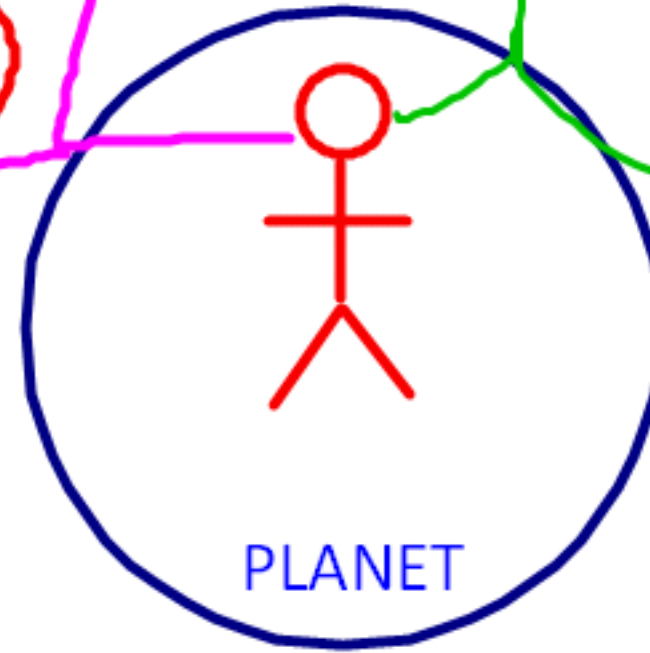


Figure 5.10 The degree of time dilation varies with velocity.

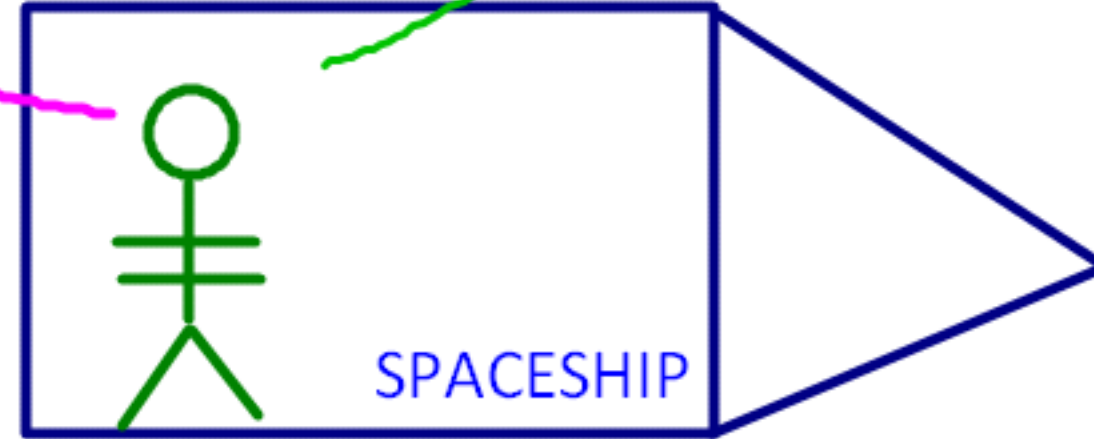
a period of
physics class to
takes 50 mins



your 2095
class
takes 35 min
for me.

for me your
physics class takes
~ 90 min

Σ



my ~~2095~~ 2095
class takes only
20 min

to 0.9c
→

2001 HSC QUESTION

Question 16 (4 marks)

Muons are very short-lived particles that are created when energetic protons collide with each other. A beam of muons can be produced by very-high-energy particle accelerators.

The high-speed muons produced for an experiment by the Fermilab accelerator are measured to have a lifetime of 5.0 microseconds. When these muons are brought to rest, their lifetime is measured to be 2.2 microseconds.

- (a) Name the effect demonstrated by these observations of the lifetimes of the muons.

- (b) Calculate the velocity of the muons as they leave the accelerator.

$$t_v > t_o$$

$$1 - \left(\frac{v}{c}\right)^2 = \left(\frac{2.2}{5}\right)^2$$

$$5 = \frac{2.2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\left(\frac{v}{c}\right)^2 = 1 - \left(\frac{2.2}{5}\right)^2$$

$$\frac{v}{c} = 0.89$$

$$v = 0.89c$$

STEPS FOR RELATIVITY PROBLEMS

1. Read and Understand the question
2. Identify the two IFRs
3. Decide what is being measured [What is the incident?]
4. Decide which IFR the incident belongs to. Label the two IFRs as "proper (rest) [t_o]" and "travelling [t_v]"
5. Check if the relative velocity of IFRs is provided.
6. Pick the appropriate formula and solve for unknown.

2008 HSC QUESTION

- 5 A spaceship is travelling away from Earth at $1.8 \times 10^8 \text{ m s}^{-1}$. The time interval between consecutive ticks of a clock on board the spaceship is 0.50 s . Each time the clock ticks, a radio pulse is transmitted back to Earth.

$$V = 1.8 \times 10^8$$

What is the time interval between consecutive radio pulses as measured on Earth?

- (A) 0.40 s
(B) 0.50 s
(C) 0.63 s
(D) 0.78 s

$$t_v = \frac{0.5}{\sqrt{1 - \left(\frac{1.8 \times 10^8}{3 \times 10^8} \right)^2}} = \frac{0.5}{\sqrt{1 - 0.6^2}} = 0.63 \text{ s}$$

STEPS FOR RELATIVITY PROBLEMS

1. Read and Understand the question
2. Identify the two IFRs
3. Decide what is being measured [What is the incident?]
4. Decide which IFR the incident belongs to. Label the two IFRs as "proper (rest) [t_o]" and "travelling [t_v]"
5. Check if the relative velocity of IFRs is provided.
6. Pick the appropriate formula and solve for unknown.

LENGTH CONTRACTION

> another result of 'constancy of speed of light'

$$\gamma = \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

↓
Lorentz factor.

$$l_v = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

\Rightarrow

$$l_0 > l_v$$

l_0 = rest length, measured on the rest frame

l_v = the measurement of the rest length from a frame moving relative to rest frame.

v : relative velocity b/w frames.

Length contraction is the shortening of an object in the direction of its motion as observed from a reference frame in r

Length contrac
called its p
reference f

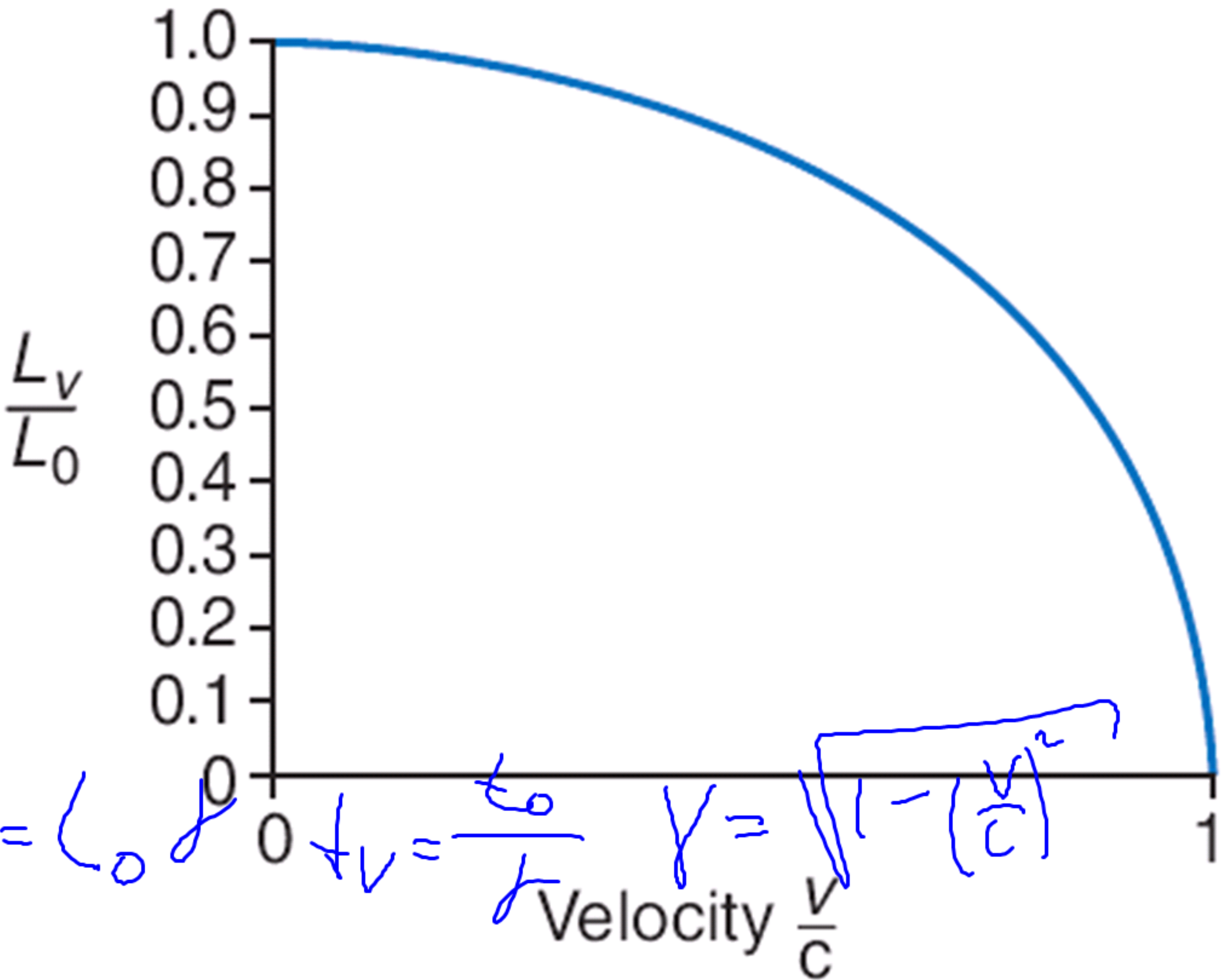
$L_v < L_o$

It can be most
Moving object

Notice that as v
the observed le

If this were a space
length, but th
planet in thei

Length contraction



Length contraction ...

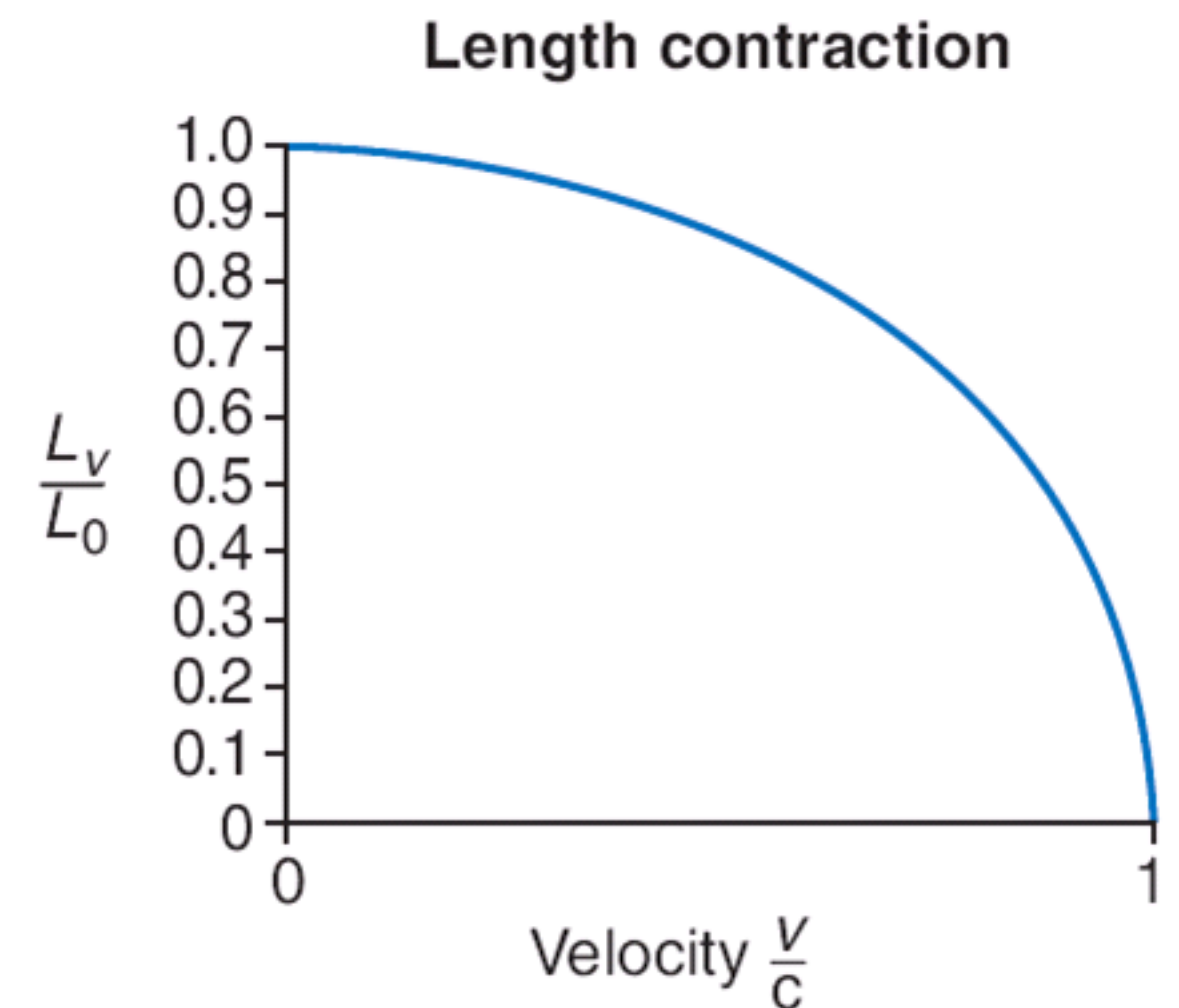
Length contraction can be generally stated as follows: the length of an object measured within its rest frame is called its proper length, L_o , or rest length. Measurements of this length, L_v , made from any other inertial reference frame in relative motion parallel to that length, are always less.

$$L_v < L_o$$

It can be most simply stated as:

...

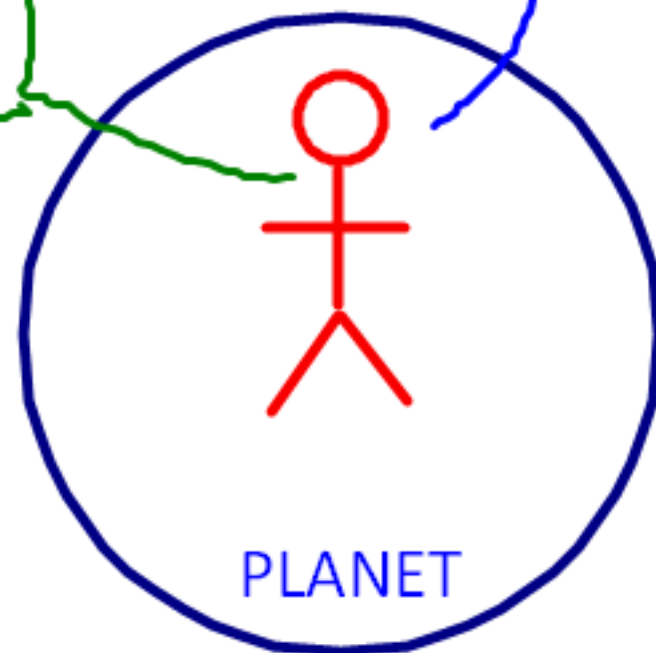
Notice that as velocity approaches the speed of light, the observed length approaches zero.



If this were a spaceship blasting past a planet at near light speed, the inhabitants of the planet would see a very short spaceship of nearly zero length, but the space travellers would notice no change at all to the length of their ship. They would, instead, briefly observe a wafer-thin planet in their windows, since from their inertial frame of reference it is the planet in rapid motion, not themselves.

earth is
12000 km
in diameter

L_0

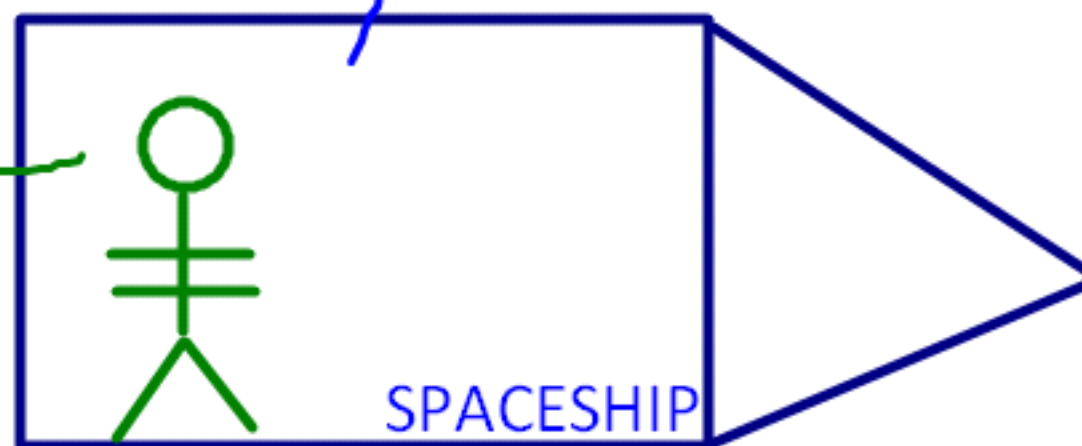


no it is
only 60m

L_v

no it is
only
8000 km

\hookrightarrow



my ship is 100m


L_0



Exercise 1: A length-contracted train

When stationary, the carriages on the state's new VVFT (very, very fast train) are each 20 m long. How long would each carriage appear to a person standing on a station platform as this express train speeds through at half the speed of light?





A diagram showing a person standing on a platform, looking at a train carriage. The person is represented by a stick figure. The train carriage is a rectangle. The platform is a trapezoid. The person is looking at the carriage from the side.

L_v $L_v = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$

$= 20 \times \sqrt{1 - \left(\frac{0.5c}{c}\right)^2}$

$= 20 \times \sqrt{0.75} = 17.32 \text{ m}$

STEPS FOR RELATIVITY PROBLEMS

1. Read and Understand the question
2. Identify the two IFRs
3. Decide what is being measured [What is the incident?]
4. Decide which IFR the incident belongs to. Label the two IFRs as "proper (rest) [t_0]" and "travelling [t_v]"
5. Check if the relative velocity of IFRs is provided.
6. Pick the appropriate formula and solve for unknown.

Exercise 2: A length-contracted person

An occupant of the VVFT looks out of a window and catches a quick glimpse of the person standing on the platform. If the thickness (from chest to back) of that person measured on the platform is 30 cm, what is the thickness observed from the train?



l_0

l_v



$$l_v = 30 \times \sqrt{1 - \left(\frac{0.5c}{c}\right)^2}$$

$$l_v = 26 \text{ cm}$$

STEPS FOR RELATIVITY PROBLEMS

1. Read and Understand the question
2. Identify the two IFRs
3. Decide what is being measured [What is the incident?]
4. Decide which IFR the incident belongs to. Label the two IFRs as "proper (rest) [t_0]" and "travelling [t_v]"
5. Check if the relative velocity of IFRs is provided.
6. Pick the appropriate formula and solve for unknown.

Exercise 3: UFOs in your backyard.

You see a flying saucer passing by Earth at $0.8c$. You measure its length to be 45 metres. What will you measure its length if it lands in your backyard.

STEPS FOR RELATIVITY PROBLEMS

1. Read and Understand the question
2. Identify the two IFRs
3. Decide what is being measured [What is the incident?]
4. Decide which IFR the incident belongs to. Label the two IFRs as "proper (rest) [t_0]" and "travelling [t_v]"
5. Check if the relative velocity of IFRs is provided.
6. Pick the appropriate formula and solve for unknown.

2007 HSC QUESTION

- 2 A spaceship sitting on its launch pad is measured to have a length L . This spaceship passes an outer planet at a speed of $0.95c$.

Which observations of the length of the spaceship are correct?

	<i>Observer on the spaceship</i>	<i>Observer on the planet</i>
(A)	No change	Shorter than L
(B)	No change	Greater than L
(C)	Shorter than L	No change
(D)	Greater than L	No change

RELATIVITY OF SIMULTANEITY

> another result of 'constancy of speed of light'

Einstein contended that if an observer sees two events to be simultaneous then any other observer, in relative motion to the first, generally will not judge them to be simultaneous.

In other words, **simultaneous events in one frame of reference are not necessarily observed to be simultaneous in a different frame of reference.**

This is known as the **relativity of simultaneity.**

RELATIVITY OF SIMULTANEITY

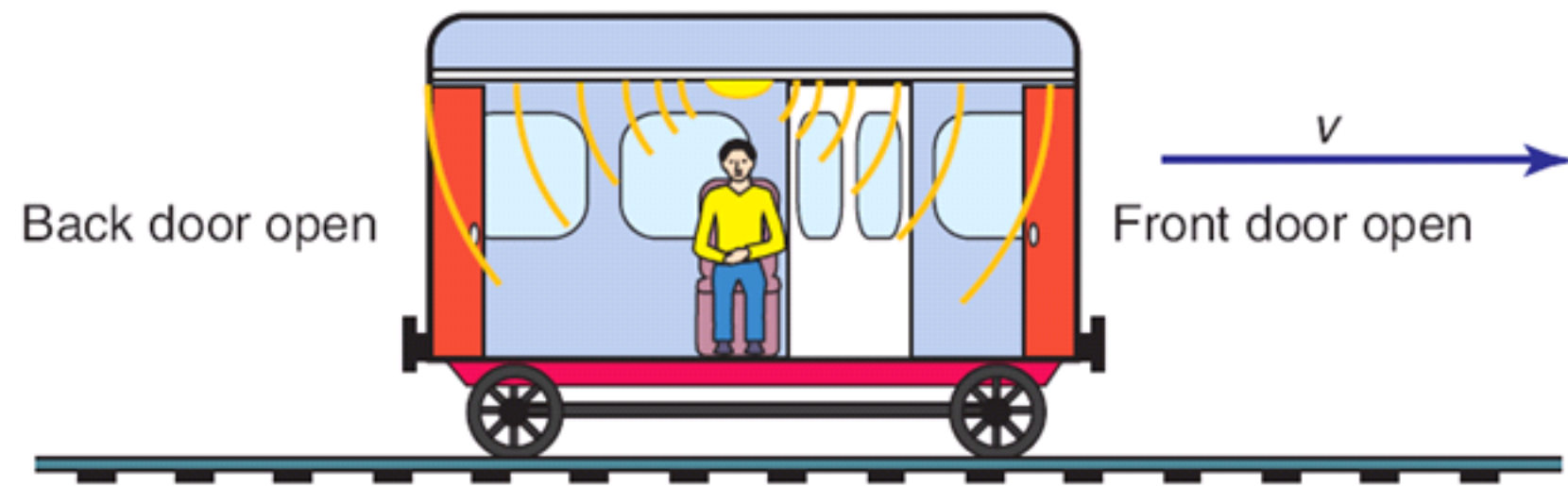
> another result of 'constancy of speed of light'

Einstein contended that if an observer sees two events to be simultaneous then any other observer, in relative motion to the first, generally will not judge them to be simultaneous.

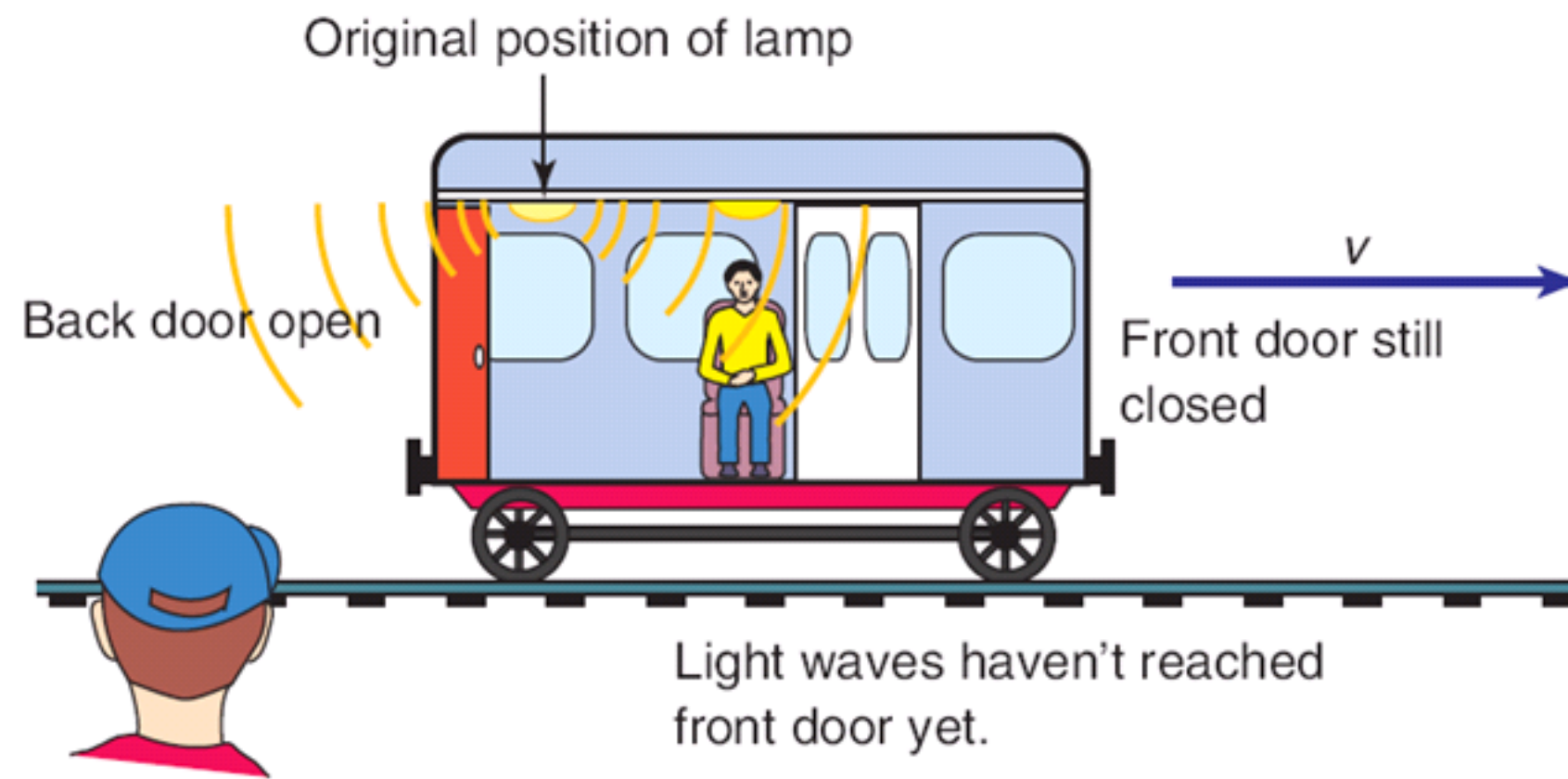
In other words, ...

...

(a) As seen by train traveller

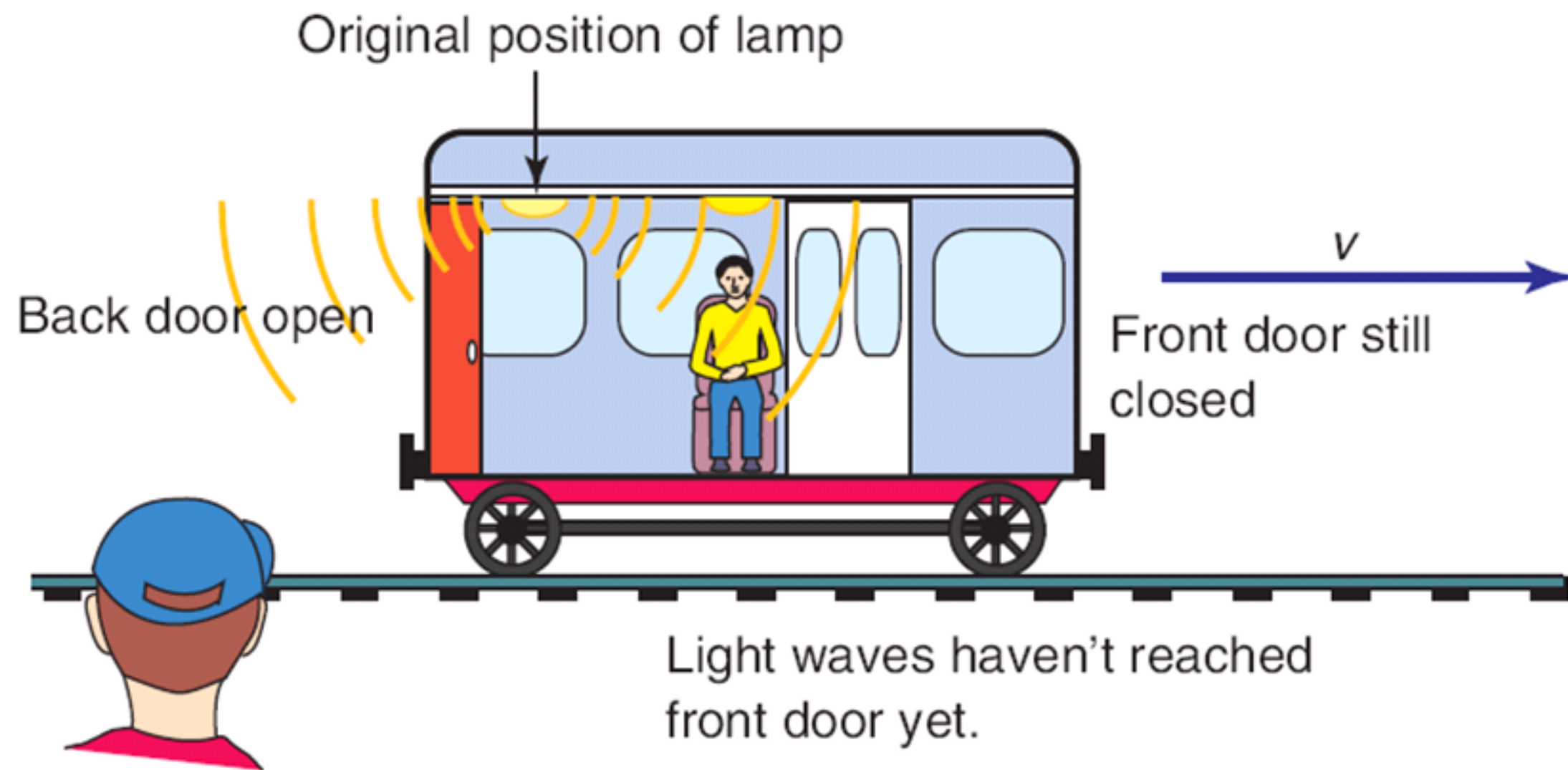


(b) As seen by stationary observer



WHAT WOULD HAPPEN IF WE TREAT LIGHT LIKE A NORMAL WAVE OR BALL AND ADD/SUBTRACT SPEED OF TRAIN TO FIND ITS NEW SPEED?

(b) As seen by stationary observer



P23 SOLUTIONS

CIRCULAR MOTION

A1.

- a** $T = 1/f$
 $= 1/2.0 \text{ s}^{-1} = 0.50 \text{ s}$
- b** $v = 2\pi fr$
 $= 2\pi(2.0 \text{ Hz})(0.80 \text{ m}) = 10.1 \text{ m s}^{-1}$
- c** $a = v^2/r$
 $= (10.053 \text{ m s}^{-1})^2/0.80 \text{ m} = 126 \text{ m s}^{-2}$ towards centre of circle
- d** $F = ma$
 $= (2.5 \text{ kg})(12.6 \text{ m s}^{-2}) = 316 \text{ N}$
- e** The force that is responsible for the centripetal acceleration of the ball is the tension in the wire, which is directed radially inwards at all times.
- f** If the wire breaks, the ball will move off at a tangent to the circle with speed 10.1 m s^{-1} .

A2.

- a** $a = v^2/r$
 $= (2.0 \text{ m s}^{-1})^2/1.5 \text{ m}$
 $= 2.7 \text{ m s}^{-2}$ towards centre of circle
- b** $F = ma$
 $= (50 \text{ kg})(2.67 \text{ m s}^{-2})$
 $= 133 \text{ N}$ towards centre of circle
- c** The horizontal force exerted on the blades by the ice; i.e. friction

A3.

- a**
- i East
 - ii North-east
 - iii South
- The direction of velocity of the ball is tangential to the circle at each point.
- b**
- i North
 - ii North-west
 - iii East
- The direction of the centripetal acceleration is radially inwards at each point.
- c**
- i West
 - ii South-west
 - iii South
- The direction of the net force at each point is the same as that of the acceleration, radially inward.
- d** West
- The direction of the velocity is tangential to the circle at that point.

A4.

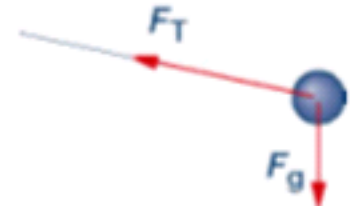
C is the correct answer. The speed of the ball is constant but the direction of the velocity, and hence the momentum, are constantly changing.

A5.

- a** $T = \frac{1}{f} = \frac{1}{1.25} = 0.800 \text{ s}$
 $v = \frac{2\pi r}{T}$
 $= 2\pi(1.00 \text{ m})/0.800 = 7.85 \text{ m s}^{-1}$
- b** $a = v^2/r$
 $= (7.854 \text{ m s}^{-1})^2/1.00 \text{ m} = 61.7 \text{ m s}^{-2}$ radially inwards

- c** $F = ma$
 $= (50 \times 10^{-3} \text{ kg})(61.7 \text{ m s}^{-2}) = 3.08 \text{ N}$ radially inwards

A6.

- a**
- 
- b** The magnitude of the tension in the wire $= F_T$, where $F_T \cos 30^\circ = 3.085 \text{ N}$ and $F_T = 3.56 \text{ N}$

A7.

$$30 \text{ km h}^{-1} = 30/3.6 = 8.3 \text{ m s}^{-1}$$

A8.

$$a = v^2/r = 8.3^2/9.2 = 7.5 \text{ m s}^{-2} \text{ towards centre of the roundabout; i.e. south}$$

A9.

- a** $\Sigma F = ma = 1200 \times 7.5 = 9.1 \times 10^3 \text{ N}$ south
- b** Friction between the tyres and the road surface

A10.

The force needed to give the car a larger centripetal acceleration will eventually exceed the maximum frictional force that could act between the tyres and the road surface. At this time, the car would skid out of its circular path.

P24 SOLUTIONS (6.3 TIME DILATION)

6.3 Time is not what it seems

1 2 s **2 a** 40 m s^{-1} **b** 0.75 s **c** 20 m s^{-1} , 1.5 s

3 a (i) 1.5 s for the backward flash and 0.75 for the forward flash **(ii)** 2.25 s **b** 0.25 s longer

4 The flashes hitting the walls are simultaneous for Anna and Ben, but not for Chloe. The flashes returning to Anna and Ben are seen as simultaneous by all observers.

5 Chloe would agree with all Anna and Ben's measurements because she would add the velocity of the train to that of the ball or sound.

6 Chloe saw the light travel at 30 m s^{-1} in her frame. Because the train was travelling in the opposite direction, she saw the light meet the back wall in a time less than that given by $30 \text{ m} / 30 \text{ m s}^{-1} = 1 \text{ s}$. In Newtonian mechanics this would have meant that Anna and Ben saw the light travel at 40 m s^{-1} , but, as we have seen, that was not the case. No one saw light travel at any speed other than 30 m s^{-1} .

7 We assumed that lengths appeared the same to Chloe as to Anna and Ben.

8 a 1 m **b** $3.3 \times 10^{-9} \text{ s}$ **c** ct_c **d** $7.6 \times 10^{-9} \text{ s}$ **e** 2.3, same

9 d $3.35 \times 10^{-9} \text{ s}$ **e** 1.005, same **10** 1.15 s

11 The equator clock is moving faster relative to poles. It is also accelerating and hence will run slower. The effect is well below what we can detect.

The Lorentz contraction factor becomes so close to one for values of v less than about $0.0001c$, that the expression

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

can't be used with a normal calculator. Fortunately, there is a simple way to find the value of γ for speeds less than about 1% of c . A binomial expansion of the term $(1 - x)^n$ tells us that, provided $x \ll 1$, $(1 - x)^n = (1 - nx)$. For the Lorentz factor, $x = (v/c)^2$ and $n = -\frac{1}{2}$, and so $\gamma = 1 + \frac{1}{2}(v/c)^2$. Thus the part of the factor greater than one can simply be found from $\frac{1}{2}(v/c)^2$.



- Our earlier simple determination of the apparent difference between Anna's and Chloe's measurements of the time for the flash of light to return to the centre of the carriage was incorrect because:
 - we made a mistake in the algebra
 - we didn't allow for the look-back time involved
 - we didn't allow for the reduced speed of light in the moving frame of reference
 - we didn't realise that the length of the carriage would be reduced because of its motion in Chloe's frame.
- If we observe a speeding rocket ship we find that (one or more):
 - its clocks seem to be going fast
 - its clocks seem to be going slow
 - its length appears to be shorter than normal
 - its length appears to be longer than normal.
- If we observe a speeding rocket ship we find that (one or more):
 - all its dimensions appear smaller
 - all its dimensions appear normal
 - its width appears smaller
 - its width appears normal.
- As Anna and Ben speed along at $0.9c$ (relative to Chloe) in their rocket ship, Anna is holding a metre rule parallel to the direction of their velocity.
 - What is the length of the rule as measured by Ben?
 - What is the length of the rule as measured by Chloe?
- Now Anna turns the ruler through a right angle so it points across the ship.
 - What is the length of the rule as measured by Ben?
 - What is the length of the rule as measured by Chloe?
- At what speed would the rocket ship be going if Chloe observed it to be half its normal length?
 - Chloe then observed the rocket ship to accelerate so that its length halved again. Did that mean that it doubled its speed? To what speed did it accelerate?
- In which of these cases will the measurement of length be the proper length?
 - Ben measures the length of his speeding rocket ship with a tape measure.
 - Chloe measures the length of Ben's speeding rocket ship from her space station.
 - Ben measures the length of Anna's speeding rocket ship from his own mini-rocket travelling at the same velocity as Anna's.
 - Chloe takes a photo of Anna's ship as it passes a fixed length scale and finds the length from the photo.
- Which one or more of the following conditions is sufficient to ensure that we will measure the proper time between two events? We must:
 - be in the same frame of reference
 - be in a frame of reference which is travelling at the same velocity
 - be stationary
 - not be accelerating with respect to the frame of the two events.
- Why is travel on a vertical line on the spacetime graph shown in Figure 6.29 an impossibility?
- When calculating the value of γ for speeds quite a bit less than that of light, it becomes difficult for a normal calculator to handle the large number of digits involved. However, as shown in the Physics file on page 217, the binomial theorem enables us to approximate the value of γ with the expression: $\gamma \approx 1 + \frac{1}{2}(v/c)^2$. Compare the values obtained using this expression with the correct values for velocities of 1%, 10%, 25% and 50% of the speed of light. At what point is the expression no longer satisfactory?
- The space shuttle orbits the Earth at around 8000 m s^{-1} . Try to find the value of gamma (γ) for the space shuttle using the correct expression, and then by using the binomial expression in Question 10.

HOMEWORK

- ✦ Homework is an integral part of your "Learning Curve", take it seriously!
- ✦ Target minimum 1 hour of Physics everyday
- ✦ Divide your physics home study in three segments;
 - ✓ Revision (past)
 - ✓ Homework (present)
 - ✓ Tomorrow (future)
- ✦ Homework is due next period, unless otherwise stated
- ✦ If you cannot do all, at least do a few from each piece

*Apart from **reading the relevant pages from the textbook and solving the rest of the questions in this booklet**
your homework is:*

- ✓ Study CSU Space 4 notes
- ✓ New questions in this booklet
- ✓ Relevant pages in Multiple Choice Dot Points Book (DPB). Bring the book for Monday.
- ✓ Space 4 Past year questions
- ✓ Chapter 5 questions 1-20

NEXT PERIOD > MASS DILATION