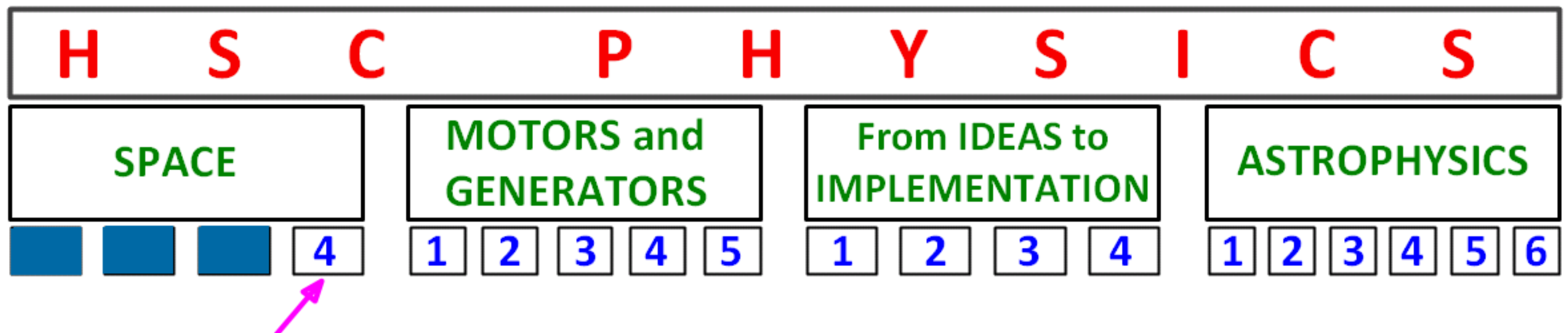


# SPACE

1<sup>st</sup> Quarter; Module 1

## PERIOD 27

Mass Dilation, Mass-Energy Equivalency



# Einstein's Theory of Special Relativity:

- ★ The laws of physics are the same in all frames of reference; that is, the principle of relativity always holds
- ★ The speed of light is independent of the motion of the observer; that is, everyone always observes the same speed of light regardless of their motion

**Also:** The luminiferous aether is superfluous (unnecessary); that is, it is no longer needed to explain the behaviour of light.

# TIME DILATION

> a result of 'constancy of speed of light'

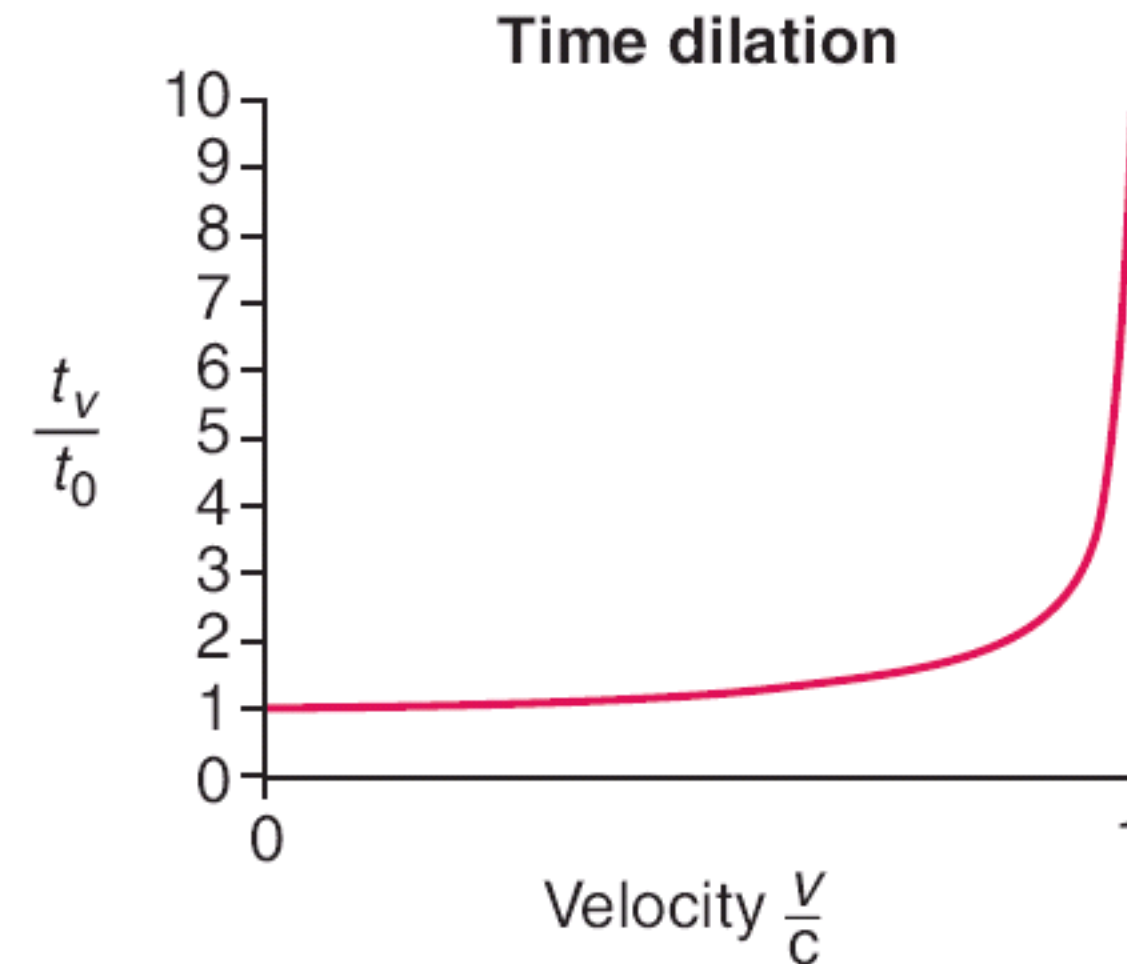
**Time dilation** is the slowing down of events as observed from a reference frame in relative motion.

**Time dilation** can be generally stated as follows: The time taken for an event to occur within its own rest frame is called the proper time  $t_0$ , or rest time. Measurements of this time,  $t_v$ , made from any other inertial reference frame in relative motion to the first, are always greater.

$$t_v > t_0$$

It can be most simply stated as: moving clocks run slow.

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$



# LENGTH CONTRACTION

## > another result of 'constancy of speed of light'

**Length contraction** is the shortening of an object in the direction of its motion as observed from a reference frame in relative motion.

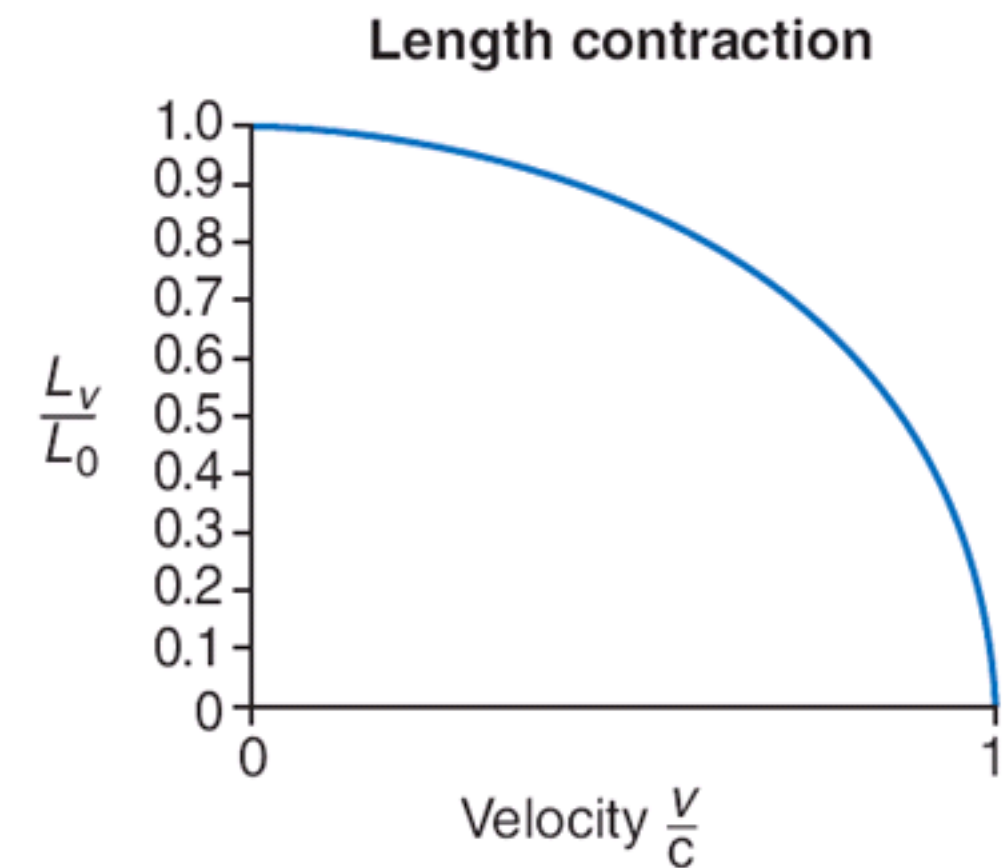
**Length contraction** can be generally stated as follows: the length of an object measured within its rest frame is called its proper length,  $L_o$ , or rest length. Measurements of this length,  $L_v$ , made from any other inertial reference frame in relative motion parallel to that length, are always less.

$$L_v < L_o$$

It can be most simply stated as:

**Moving objects shorten in the direction of their motion.**

$$l_v = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$



Notice that as velocity approaches the speed of light, the observed length approaches zero.

# RELATIVITY OF SIMULTANEITY

> another result of 'constancy of speed of light'

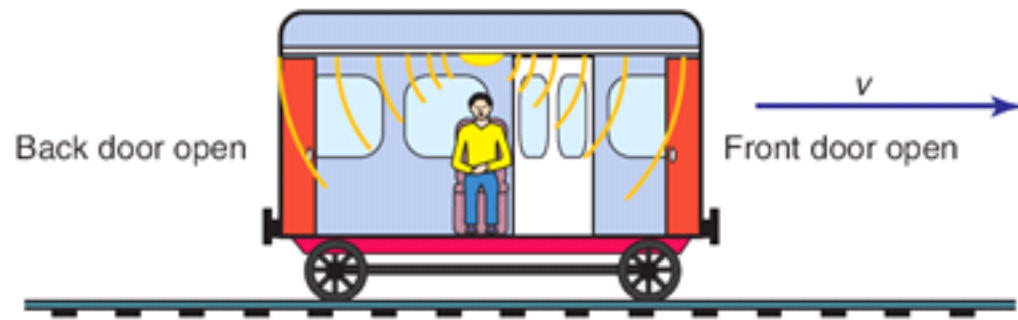
Einstein contended that if an observer sees two events to be simultaneous then any other observer, in relative motion to the first, generally will not judge them to be simultaneous.

In other words, **simultaneous events in one frame of reference are not necessarily observed to be simultaneous in a different frame of reference.**

This is known as the **relativity of simultaneity.**

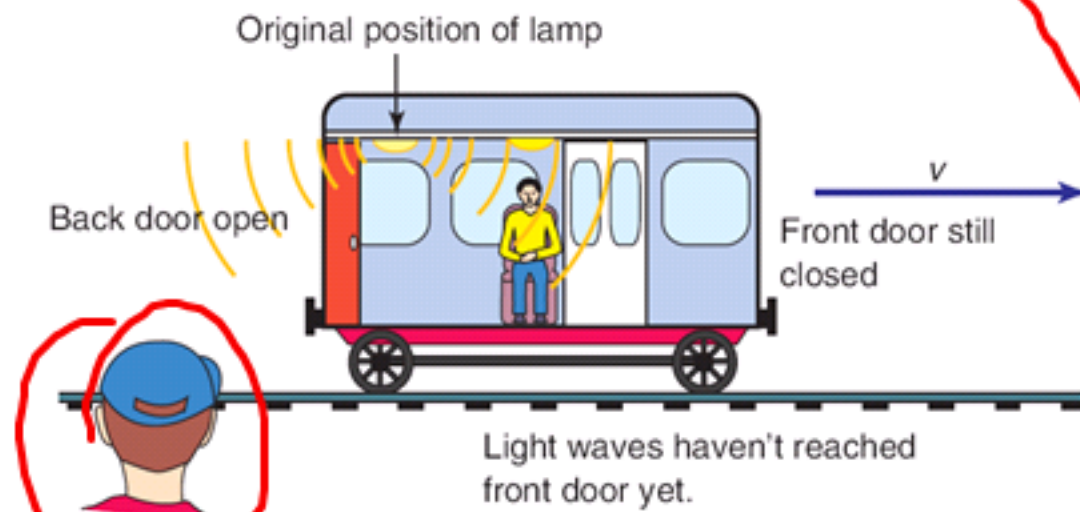


(a) As seen by train traveller



he sees  
both doors  
open simultaneously

(b) As seen by stationary observer



she sees back  
door open first,  
then front door.

simultaneity  
is

relative

"due to the  
fact that  
speed of light  
is constant."

## DEFINITION OF "METRE"

- ✦ The metre as a unit of length was first defined in 1793 when the French government decreed it to be  $1 \times 10^{-7}$  times the length of the Earth's quadrant passing through Paris.
- ✦ When it was discovered that the quadrant survey was incorrect, the metre was redefined as the distance between two marks on a bar.
- ✦ In 1875 the Systeme Internationale (SI) of units was set up so that the definition became more formal: a metre was the distance between two lines scribed on a single bar of platinum–iridium alloy.
- ✦ There is always a need for the accuracy of a unit of measure to keep pace with improvements in technology and science, so the metre has since been redefined twice.

## CURRENT DEFINITION OF METRE

- ✦ The current definition of the metre uses the constancy of the speed of light in a vacuum (299 792 458 m/s) and the accuracy of the definition of one second (9 129 631 770 oscillations of the 133 Cs atom), to achieve a definition that is both highly accurate and consistent with the idea of space–time.
- ✦ One metre is now defined as the length of the path travelled by light in a vacuum during the time interval of  $1/299\,792\,458$  of a second.
- ✦ The term 'light-year' is a similar distance unit, being the length of the path travelled by light in a time interval of one year. One light-year is approximately equal to  $9.467\,28 \times 10^{12}$  km.



## A STRONG SUPPORTER OF TIME DILATION - MUONS

One interesting proof of relativity came in the observation of subatomic particles called *muons* created by collisions of cosmic rays with atoms high in the Earth's atmosphere.

- ✓ Muons have a very short life. On average, half of them only live for 2.5 microsecond ( $10^{-6}\text{s}$ ).
- ✓ It is easy enough to measure their speed as they plunge down through the atmosphere—it is about  $0.99c$ , almost the speed of light.
- ✓ Now at that speed, a muon that lives 2.5 microsecond will travel  $3 \times 10^8 \times 2.5 \times 10^{-6} = 750 \text{ m}$ , not nearly far enough to reach the surface of the Earth, typically 15 km below.
- ✓ At that speed it would take close to 50 microsecond to reach the ground.
- ✓ However, over 10% of the muons created at that height *do reach* the ground. The explanation, as you have probably guessed, is that time is passing much more slowly for the muons-as we see it.
- ✓ At  $0.99c$ , the Lorentz factor is 7.1 and so the muon half-life (in our scheme of things) is nearly 18 microsecond.
- ✓ This is still not long enough for the average muon to reach ground level, but a significant number of muons (12.5%) will live for three half-lives, which *is* long enough to reach the ground.



# MASS DILATION > another result of 'constancy of speed of light'

**Mass dilation** is the increase in the mass of an object as observed from a reference frame in relative motion.

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$m_0$  = mass measured in the rest frame of reference, rest/proper mass

$m_v$  = mass as seen from the frame of reference in relative motion to the rest frame

$v$  = relative velocity of the frames

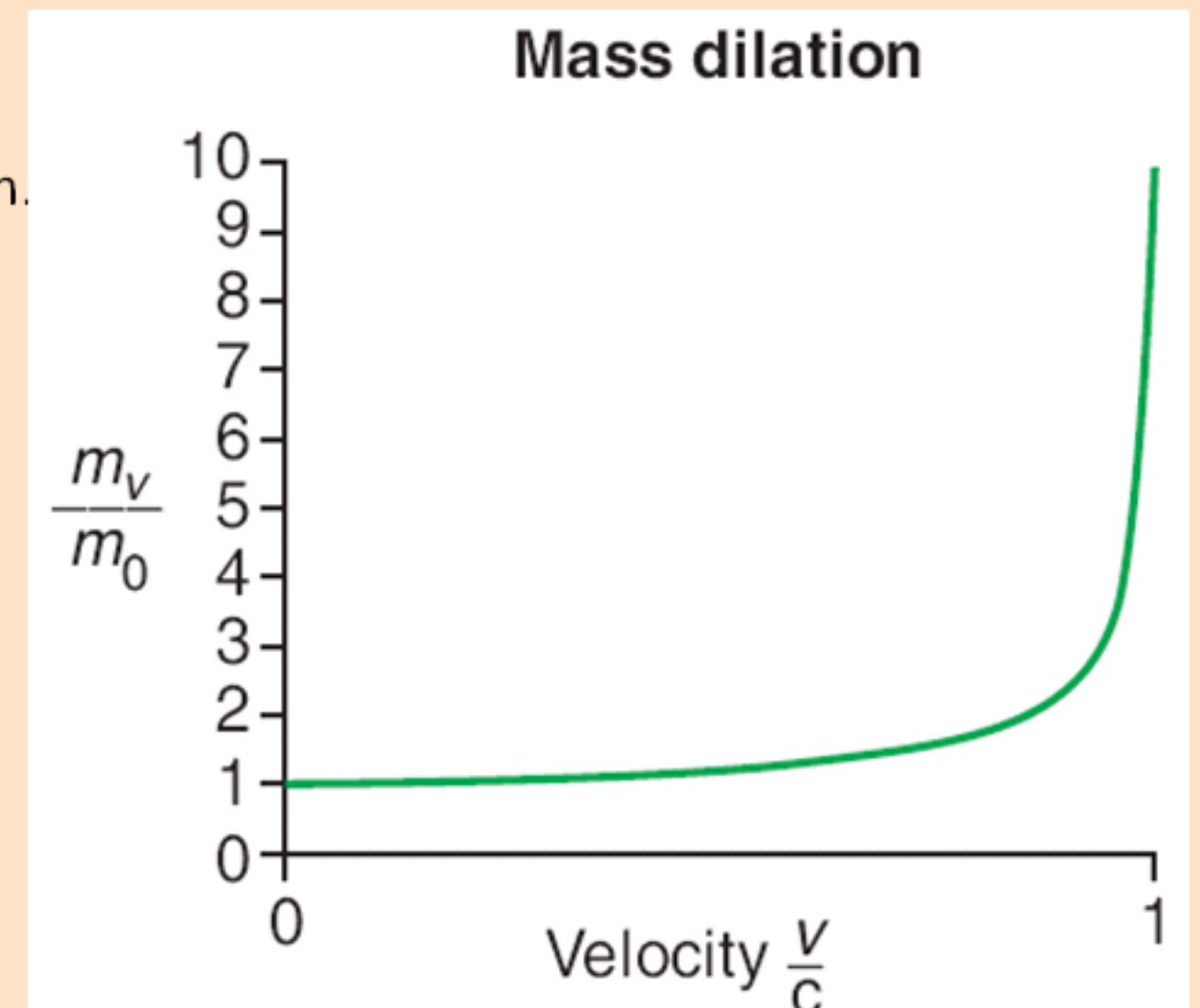
$c$  = speed of light

**Mass dilation** can be stated as follows: The mass of an object within its own rest frame is called its rest mass  $m_0$ . Measurements of this mass  $m_v$ , made from any other inertial reference frame in relative motion to the first, are always greater.

$$m_v > m_0$$

The effect can be most simply stated as: Moving objects gain mass.

The degree of mass dilation varies with velocity as shown in the graph.



# MASS DILATION > another result of 'constancy of speed of light'

## Mass dilation \*

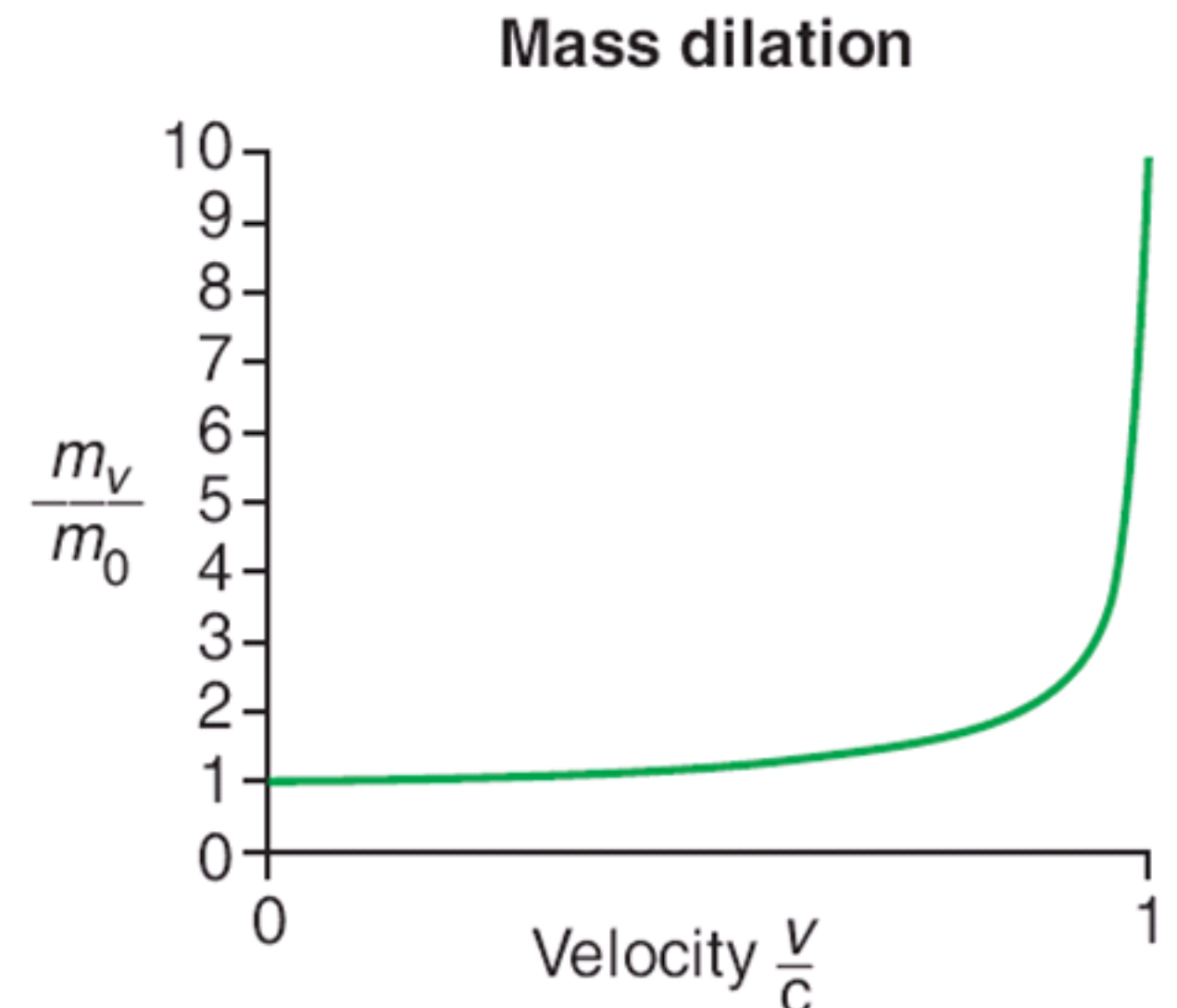
$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**Mass dilation** can be stated as follows: The mass of an object within its own rest frame is called its rest mass  $m_0$ . Measurements of this mass  $m_v$ , made from any other inertial reference frame in relative motion to the first, are always greater.

$$m_v > m_0$$

The effect can be most simply stated as: \*

The degree of mass dilation varies with velocity as shown in the graph.



## Experimental evidence for mass dilation

- ✦ In 1909 it was noticed that beta particles (electrons) emitted by different radioactive substances possessed different charge to mass ratios ( $q/m$ ). The various particles were travelling at significant fractions of the speed of light. Furthermore, the greater the speed of the beta particle, the smaller was its charge:mass ratio (since  $m$  increases). When the effect of mass dilation was accounted for (when  $m_0$  is calculated from  $mv$  and speed), the beta particles were all found to have the same charge:mass ratio.
- ✦ Modern particle accelerators, however, demonstrate mass dilation every time they are used. As they accelerate particles, such as electrons or protons, to relativistic speeds, ever greater forces are required as the particles' masses progressively increase.
- ✦ The electrons in a cathode ray TV set are actually travelling at around 30% of the speed of light and the magnetic deflection that paints the picture on the screen would give us a very fuzzy and small picture if the relativistic effects on the deflection were not taken into account.



**Figure 6.35** The relativistic increase in mass of the electrons in a TV tube needs to be taken into account by the engineers who design TVs or the picture would be small and distorted.



# 3 Golden Steps in Relativity Problems

**1** - Identify the (inertial) Frame of References; **there must be two**

**2** - (Once you identify the frame of references) decide which one is **Rest frame** ( $t_o, L_o, m_o$ ) and which one is **Travelling frame** ( $t_v, L_v, m_v$ )

*Ask yourself "What am I measuring/recording?" and "from where?"*

**3** - Choose the appropriate formula and solve it for unknown



### Exercise 1: The rest mass of an electron

The rest mass of an electron is  $9.109 \times 10^{-31}$  kg. Calculate its mass if it is travelling at 80 per cent of the speed of light.

$$m_0 = 9.109 \times 10^{-31} \text{ kg}$$

$$v = 0.8c$$

$$m_v = \frac{9.109 \times 10^{-31}}{\sqrt{1 - \left(\frac{0.8c}{c}\right)^2}} = 15 \times 10^{-31} \text{ kg}$$

$$m_v = 1.$$

$$1\% \quad m_v = 1.01 m_0$$

$$0.37\% \quad \boxed{m_v = 1.0037 m_0}$$

$$m_v = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$1.0037 m_0 = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$1 - \left(\frac{v}{c}\right)^2 = \left(\frac{1}{1.0037}\right)^2$$

$$\sqrt{1 - \left(\frac{1}{1.0037}\right)^2} = \sqrt{\left(\frac{v}{c}\right)^2}$$

$$v = \sqrt{1 - \left(\frac{1}{1.0037}\right)^2} \times c$$

$$v = 0.085c$$

- (c) A mass is moving in an inertial frame of reference at a velocity  $v$  relative to a stationary observer. The observer measures an apparent mass increase of 0.37%.

3

Calculate the value of  $v$  in  $\text{m s}^{-1}$ .

# WHY IS IT IMPOSSIBLE TO REACH THE SPEED OF LIGHT?

As the speed of an object approaches the speed of light  $c$ , its mass approaches an infinite value. It is this enormous increase in mass that prevents any object from exceeding the speed of light. This is because;

- ✦ An applied force is required to create acceleration.
- ✦ Acceleration leads to higher velocities, which eventually leads to increased mass.
- ✦ This means that further accelerations will require ever greater force.
- ✦ As mass becomes infinite, an infinite force would be required to achieve any acceleration at all.
- ✦ Sufficient force can never be supplied to accelerate beyond the speed of light.

Handwritten red annotations:

- An arrow points from the first bullet point to the equation  $a = \frac{F}{m}$ .
- Next to the second bullet point, it says "as  $v \uparrow$  so does 'm'".
- Below that, it says "F!" with an exclamation mark.
- At the bottom, it shows the equation  $a = \frac{F}{m}$  again, with the 'm' in the denominator written in a larger, more stylized script.

# WHY IS IT IMPOSSIBLE TO REACH THE SPEED OF LIGHT?

As the speed of an object approaches the speed of light  $c$ , its mass approaches an infinite value. It is this enormous increase in mass that prevents any object from exceeding the speed of light. This is because;

✦ .

✦ .

✦ .

✦ .

✦ .



## ***THE EQUIVALENCE OF MASS AND ENERGY***

But herein lies a problem:

- ✦ If a force is applied to an object, then work is done on it. Another way to say this is that energy is given to the object.
- ✦ This energy would take the form of increased kinetic energy as the object speeds up.
- ✦ But at near light speed the object does not speed up as we would normally expect

### **SO WHERE IS THE ENERGY GOING?**

- ✦ The applied force is giving energy to the object and the object does not acquire the kinetic energy we would expect.
- ✦ Instead, it acquires extra mass. Einstein made an inference here and stated that the mass (or inertia) of the object contained the extra energy.

## ***THE EQUIVALENCE OF MASS AND ENERGY***

But herein lies a problem:

- ✦ If a force is applied to an object, ...

Another way to say this is that energy ...

- ✦ This energy would take the form of ...
- ✦ But at near light speed the object ...

### **SO WHERE IS THE ENERGY GOING?**

- ✦ The applied force is ...

- ✦ Instead, ...

Einstein made an inference here and stated that ...



## 5.7

### *The rest energy of an electron*

What is the energy equivalent of an electron of mass  $9.109 \times 10^{-31}$  kg?



**Estimate the energy equivalent of the mass of a granny smith apple**

Find the power rating of our SUN, if 564 million tonnes of Hydrogen turns into 560 million tonnes of Helium every second.



# P27

## H/W

"Solutions of P26, P27 and P28 will be given with P29"

- 11 Spaceships A and B leave the Earth and travel towards Vega, both at a speed of  $0.9c$ . Observer C back on Earth sees the crews of A and B moving in 'slow motion'. Describe how the crew of A see the crew of B, and how they see C and the Earthlings moving.
- A B will appear normal, C will be sped up.
  - B B will appear normal, C will be slowed down.
  - C B will appear slowed down, C will be normal.
  - D B will appear sped up, C will be slowed down.
  - E None of these.
- 12 One of the fastest ever objects ever made on Earth was the Galileo Probe which, as a result of Jupiter's huge gravity, entered its atmosphere in 1995 at a speed of nearly  $50\,000\text{ m s}^{-1}$ . Which of the following is the best estimate of the Lorentz factor for the probe? (You may wish to use the expression  $\gamma \approx 1 + v^2/2c^2$ .)
- A Less than 1
  - B 1.000 000 00
  - C 1.000 000 01
  - D 1.1

The following information applies to questions 13 and 14.



Ben and Chloe are playing table tennis in their space ship as they rush past Anna in her space station at a relative speed of  $240\,000\text{ km s}^{-1}$ . Ben and Chloe say that they are hitting the ball back and forth with a frequency of 1 hit per second. Their table is 3.0 m long and 1.0 m high.

- 13 Which of the following is the best estimate of the time between hits as measured by Anna?
- A 0.7 s
  - B 0.8 s
  - C 1.0 s
  - D 1.7 s
- 14 How long and high is the table as seen by Anna?
- A 3.0 m long, 1.0 m high
  - B 3.0 m long, 0.7 m high
  - C 1.8 m long, 1.0 m high
  - D 1.8 m long, 0.7 m high
- 15 If a spaceship is travelling at 99% of the speed of light, which of the following best explains why it can't simply turn on its engine

- and accelerate through and beyond the speed of light,  $c$ —as the increase in momentum should be equal to the impulse applied?
- A The law of impulse equals change of momentum does not apply at speeds close to  $c$ .
  - B While the momentum increases with the impulse, it is the mass rather than the speed that is getting greater.
  - C The spaceship does actually exceed  $c$ , but it doesn't appear to from another frame of reference because of length contraction of the distance it covers.
  - D Given enough impulse the spaceship could exceed  $c$ , but no real spaceship could carry enough fuel.
- 16 Physicists sometimes say that the mass of an electron is about  $8 \times 10^{-14}\text{ J}$ . Which of the following best explains what is meant by this statement?
- A This is the 'rest energy' of the electron which, as Einstein showed, is equivalent to the mass.
  - B This is a misprint; it should be  $8 \times 10^{-14}\text{ kg}$ .
  - C This is a shorthand way of saying that if all the mass of an electron was converted to energy, we would get this amount of energy.
  - D This is the energy of an electron which is travelling at the speed of light.

In the following questions, assume that phrases such as 'high speed' mean speeds at a significant fraction of the speed of light. In some questions you may need to use the binomial approximation for the Lorentz factor:  $\gamma \approx 1 + v^2/2c^2$ .

- 17 Make a brief comment on the importance or otherwise, to Einstein's theory, of each of the alternatives A–D in Question 6.
- 18 Make a brief comment on the correctness or otherwise of each of the alternatives A–D in Question 7.
- 19 Aristotle suggested that the 'natural' state of motion for any object is rest. Galileo introduced the principle of inertia which suggested that the natural state of motion is constant velocity (zero velocity being just one example). Explain why Aristotle's view was so hard to shake, and why, if we had spent time as an astronaut in a space station, Galileo's principle would be much easier to accept.
- 20 Maxwell actually felt that there was some sort of error in his equations that predicted the speed of light. Describe the supposed error and the reasons Maxwell was convinced there was a problem with the equations.
- 21 The Michelson–Morley experiment was an attempt to measure the speed of the Earth through the aether.
- a Describe the basic assumption upon which this attempt was based.
  - b Why did they use an apparatus which compared the speed of light in perpendicular directions?



# **HOMEWORK**

- ✦ Homework is an integral part of your "Learning Curve", take it seriously!
- ✦ Target minimum 1 hour of Physics everyday
- ✦ Divide your physics home study in three segments:
  - ✓ Revision (past)
  - ✓ Homework (present)
  - ✓ Tomorrow (future)
- ✦ Homework is due next period, unless otherwise stated
- ✦ If you cannot do all, at least do a few from each piece

*Apart from **reading the relevant pages from the textbook and solving the rest of the questions in this booklet**  
your homework is:*

- ✓ Personal Notes - at least 5 pages every day
- ✓ New booklet
- ✓ Study CSU Space 4 notes
- ✓ New questions in this booklet
- ✓ Relevant pages in Multiple Choice Dot Points Book (DPB). Bring the book for Monday.
- ✓ Space 4 Past year questions
- ✓ Chapter 5 questions 1-20

**NEXT PERIOD > MASS-ENERGY EQUIVALANCE, SPACE TRAVEL**